Crash Risk in Currency Markets [*]

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Abstract

Since the Fall of 2008, out-of-the money puts on high interest rate currencies have become significantly more expensive than out-of-the-money calls, suggesting a large crash risk of those currencies. To evaluate crash risk precisely, we propose a parsimonious structural model that includes both Gaussian and disaster risks and can be estimated even in samples that do not contain disasters. Estimating the model for the 1996 to 2014 sample period using monthly exchange rate spot, forward, and option data, we obtain a real-time index of the compensation for global disaster risk exposure. We find that disaster risk accounts for more than a third of the carry trade risk premium in advanced countries over the period examined. The measure of disaster risk that we uncover in currencies proves to be an important factor in the cross-sectional and time-series variation of exchange rates, interest rates, and equity tail risk.

Currency carry trades are investment strategies where an investor borrows in low-interest rate currencies and invests in high-interest rate currencies. Such simple strategies offer large expected excess returns, challenging the benchmark models in international macroeconomics. In this paper, we explore whether crash risk can explain these excess returns. Crash risk is driven by rare but large adverse aggregate shocks to stochastic discount factors. The small number of such shocks in the samples examined in the macroeconomics and finance literature is a major difficulty in the estimation of the compensation for crash risk. To address this difficulty, we turn to currency option markets.

Currency options reveal a stark contrast between the pre- and post-2008 crisis periods. As we shall see, before the Fall of 2008, G10 option prices were only mildly asymmetric across strikes, with small differences between the price of an out-of-the-money put — an insurance against large depreciations — and the price of an out-of-the-money call — an insurance against large appreciations. During the Fall of 2008, however, high interest rate currencies sharply depreciated while low interest rate currencies appreciated. Carry traders borrowing in Japanese yen and lending in New Zealand dollars lost close to 30% of their investment in October 2008. Since the Fall of 2008, there have been significant differences between high and low interest rate currencies in the currency option markets. Out-of-the money puts on high interest rate currencies have become more expensive than out-of-the-money calls, indicating a high risk of large depreciations in those currencies, which contrasts with the low risk of depreciation of the low interest rate currencies.

The Fall of 2008 thus appears as a defining moment for the currency markets of developed countries, recalling the 1987 crisis for equity markets: before 1987, equity option smiles are non-existent, while after 1987, they became central to equity option markets, pointing towards deviations from the lognormality assumption of the Black and Scholes (1973) option pricing formula. After 2008, currency options prices are clearly asymmetric, especially for high interest rate currencies.

Against this empirical background, we propose a parsimonious exchange rate model and a simple methodology using currency option prices to estimate the compensation for global disaster risk exposure even in samples without disasters. We find that, in our sample, the compensation for

disaster risk exposure is statistically significant and accounts for more than a third of average carry trade excess returns in the G10 currencies.

In our model, financial markets are complete and thus the log change in the exchange rate is the log difference between the domestic and foreign stochastic discount factors (SDFs). Following Backus, Foresi, and Telmer (2001), we write the law of motion of the SDF in each country. These SDFs incorporate both a traditional log-normal component, as in Lustig, Roussanov, and Verdelhan (2011), and a disaster component, as in Du (2013) and Farhi and Gabaix (2013). The former responds to random shocks observed every period, while the latter responds to rare global disaster shocks that affect countries differently.

Our notion of crash risk is inclusive, as we do not specify preferences. Our setup therefore encompasses models with very large consumption growth jumps à la Rietz (1988) and Barro (2006), but also long-run risk models with more moderate macroeconomic jumps (Drechsler and Yaron, 2011), as well as models that are more agnostic about the economic origins of jumps (Bates 1991, 1996). These models all generate large, left-skewed distributions of returns under the risk-neutral measure, a feature they share with some models of stochastic volatility [in which volatility goes up during bad economic periods, as in Heston (1993) or Bates (2012)]. This common feature is our focal point. Our objective is not to discriminate among these models, which can generate comparable risk-neutral distributions at the one-month frequency. Instead, our goal is to measure the importance of the non-Gaussian shocks that most exchange rate models in the tradition of Obstfeld and Rogoff (1995) overlook.

For carry trade investors, the change in the exchange rate over the investment period is the sole source of risk. If investment currencies depreciate or funding currencies appreciate, then investors' returns decrease because they lose on their investment or must reimburse larger amounts. In our model, such exchange rate movements can be due to the usual Gaussian shocks, or to more extreme disaster shocks. The size of the disaster impact in each country is a stochastic variable; at the start of each month, the magnitudes of potential disasters are unknown. In the spirit of the macro-finance literature on disaster risk (Brunnermeier, Nagel, and Pedersen, 2008; Burnside et al., 2011;

Wachter, 2013; and Seo and Wachter, 2015), we abstract from daily variation in exchange rate volatility and volatility risk premia, but allow volatility to freely change every month. Our model delivers closed-form solutions for call and put option prices in- and out-of-the-money, as well as expected currency excess returns when the investment horizon tends towards zero. Conditional on no disaster in the sample, the expected currency excess returns are simply the sum of the compensation for Gaussian and disaster risks. While simple enough to be solved in closed form, our model is rich enough to replicate and interpret the key findings of Brunnermeier, Nagel, and Pedersen (2008) on carry trades.

We turn to currency data to estimate the compensation of disaster risk at each point in time and to test the model's implications. The data set comprises monthly currency spot, forward, and option contracts collected by J.P.Morgan for the 10 most developed currency markets. The data set starts in January 1996 and ends in August 2014. Fall 2008 can be interpreted either as a period of financial disaster, or a period of moderate consumption disaster. Alternatively, it could be interpreted as a period when there is a sharp increase in the probability of a macroeconomic disaster, but not a full-blown disaster. Our estimates of the compensation for disaster risk exposure do not depend on such interpretation, and we report them both for samples that include or exclude this period. We assume that the model parameters are constant over one month, but can vary non-parametrically from one month to the next. The model thus allows for monthly time variation in the expected exchange rate volatility, as well as changes in the disasters' probabilities and sizes.

In order to focus on carry trade risk, we sort currencies by their interest rates into three portfolios, as in Lustig and Verdelhan (2007). The average excess return on the highest interest rate currencies is large and significant at 4.3% and thus is our benchmark currency risk premium. The model parameters are estimated from the option prices of the five most liquid strikes. Currency option markets offer the perfect setting to measure the price of global disaster risk for three reasons: they are among the most liquid and developed option markets in the world; exchange rates offer a direct measure of the pricing kernels, without any assumption on aggregate cash flows; and carry trade risk is a compensation for global, not local, shocks. The minimization between the model-implied

option prices and the market prices allows us to estimate the model's parameters of interest for all currencies jointly at each date in the sample. The estimation procedure then delivers a time series of the compensation for world disaster risk. To the best of our knowledge, this time series is the first estimation of investor compensation for global disaster risk.

On average over the whole sample, excluding the Fall of 2008, investors who bear disaster risk on currency markets received a compensation for disaster risk exposure of 2.3%. Consistent with the evidence on currency option smiles, the compensation for disaster risk increases a lot post-crisis. Although expected volatility is now back to its pre-crisis level, the price of disaster risk is still an order of magnitude higher than before the crisis. The large role of disaster risk is a robust finding: the inclusion of transaction costs leads to similar results, and the absence of counterparty risk in the analysis actually suggests that disaster risk might be even more important than estimated here.

We therefore present a simple structural estimation of the compensation for global disaster risk exposure. The model is parsimonious and flexible; however, despite its flexibility, it delivers closed-form expressions for the key object of interests. The closed-form expressions then lead to a simple, transparent, and easily replicable estimation procedure that takes into account the common parameters in all currency pairs. Such strengths come with a price. In the model, we assume that Gaussian shocks are jointly normal and independent of the disasters, an assumption that is not directly testable with changes in exchange rates, as they pertain to differences in shocks, not country-specific shocks. In the data, we estimate a time series of "disaster risk exposure," which corresponds to the expected returns of carry trades in samples without disasters, not the disaster risk premium, which includes both the disaster exposure, as well as the expected return of carry trades in times of disasters. The latter is naturally difficult to estimate, as macroeconomic disasters, unlike small jumps at high frequencies, are rare. Indeed, our sample of monthly changes in exchange rates contains only one disaster-like behavior, the Fall of 2008. The exchange rate variation in 2008 and the consensus estimate of the average disaster probability in the macro-finance literature imply that the difference between the disaster risk exposure and the disaster risk premium is 0.7% on average. As a result, the disaster risk premium of 1.6% (2.3% minus 0.7%) accounts for more than

one-third of the 4.3% carry trade risk premium. The disaster risk exposure is, however, the key object of interest: in disaster-based equilibrium models of exchange rates (e.g., Farhi and Gabaix, 2013), where the exchange rate is the expected present value of future fundamentals, the disaster exposure determines the value of a currency, rather than the disaster risk premium per se. Consistent with the macro-finance literature (e.g., Barro, 2006; Gourio, 2008; Wachter, 2013), we focus on predictions that hold in samples without disasters and study the time series and cross-sectional dynamics of the disaster risk exposure.

The model and its estimation capture first-order economic links between interest rates, exchange rates, and disaster risk. First, the model implies a strong link between interest rates and the exposure to disaster risk. In the model, interest rates depend on the drift of the SDF and the exposure to disaster risk: interest rates are high in countries whose currencies tend to depreciate when disasters occur. In the data, we find a strong link between the average compensation for disaster risk implied in currency options and the average interest rates. Figure 1 reports the average estimated investor compensation for disaster risk, as well as the average interest rate differential for each country. Clearly, they align. This figure echoes the first figure in Brunnermeier, Nagel, and Pedersen (2008), who were the first to show that risk-reversals increase with interest rates. Our work builds on their findings, and the estimation of our model, which disentangles Gaussian from non-Gaussian shocks, suggests that a large part of the cross-country differences in interest rates corresponds to different exposures to global disaster risk.

[Figure 1 about here.]

Second, the model implies that countries with small (large) exposures to global disaster risk should depreciate (appreciate) during disasters (or when disaster probability increases). This is the key risk that carry traders face and the core mechanism of the model. As Figure 2 shows, this is exactly what happened during the Fall of 2008. Countries with estimated low risk exposure depreciated, while those with estimated large risk exposure appreciated. The strong link between disaster's exposures and changes in exchange rates during that period appears whether disaster

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exposure is measured during the Fall of 2008 or in the months preceding it (e.g., from May 2008 to August 2008).

[Figure 2 about here.]

Figures 1 and 2 provide strong support for the key mechanism and implications of the model. The model, however, could be easily rejected by additional data: the model ignores any potential market segmentation between currency markets and other asset markets; it does not attempt to model the full term structure of interest rates; it does not describe cash flows nor equity returns; and it is written and used at a monthly frequency and ignores daily or intra-day exchange rate variation. The model could be extended in many dimensions, but we focus instead on its core and use it to uncover some new links across asset markets and to reinterpret some recent results in the literature.

Importantly, our novel and country-specific time-series of disaster risk exposures are significantly correlated to the dynamics of short-term interest rates, exchange rates, and equity risk. When a given currency's disaster risk increases, contemporaneously three things happen: (i) its interest rate increases, (ii) its currency depreciates, and (iii) the disaster risk in its equity market (as measured by equity risk reversals) increases. We document those three patterns in panel regressions with country fixed effects. Similar results are obtained at the country- and portfolio-levels. Disaster risk appears as an important factor accounting for the cross-sectional and time-series variation of exchange rates and interest rates. These facts are consistent with disaster models of exchange rates such as Farhi and Gabaix (2013).

Finally, we derive closed-form expressions for hedged currency excess returns when the investment horizon tends toward zero. Hedged strategies protect investors against large exchange rate changes of two types: those due to disasters and those that might occasionally happen in a world of Gaussian shocks. We show that, in the limit of small time horizons, expected hedged currency excess returns are thus equal to a fraction of the Gaussian risk exposure, which varies with the put option strike used to hedge the investment. The result is intuitive: if the option strike is far from

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the money, the investor bears a large amount of depreciation risk before the option contract pays off and delivers any insurance, and thus the investor expects a large return on the hedged carry trade as a compensation for this exchange rate risk. We show, however, that disaster risk cannot be fully hedged with a simple put option when the time horizon is not negligible. Therefore, average hedged currency excess returns offer only a biased estimation of disaster risk exposure.

The paper is organized as follows. Section 1 compares the currency option smiles pre- and post-2008 for high versus low interest rate currencies. Section 2 presents our model and derives the estimation procedure. Section 3 reports our estimation of time-varying disaster risk exposure. Section 4 studies the contemporaneous links between interest rates, exchange rates and equity risk on the one hand, and disaster risk on the other hand. Section 5 derives additional results on hedged currency excess returns and risk-reversals. Section 6 reviews the literature. Section 7 concludes. The Online Appendix details all the mathematical proofs and reports additional simulation and estimation results.

1   Currency Option Smiles Pre- and Post-Crisis

We first describe our data, define some useful option-related terms, and then compare currency option smiles pre- and post-crisis.

1.1   Spot and Forward Exchange Rates

Our data set comes from J.P.Morgan and focuses on the 10 largest and most liquid currency spot, forward, and option markets: Australia, Canada, Euro area, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom, and United States. All exchange rates in our sample are expressed in U.S. dollars per foreign currency. As a result, an increase in the exchange rate corresponds to an appreciation of the foreign currency and a decline of the U.S. dollar. For each currency, the sample comprises spot and one-month forward exchange rates measured at the end of the month, as well as implied volatilities from currency options with one-month maturity for the same dates. Foreign

interest rates are built using forward currency rates and the U.S. LIBOR, assuming that the covered interest rate parity condition holds.[1]

1.2   Option Lexicon

Before turning to our option data, let us review some basic option terms. Figure 3 presents the payoffs of the three option-based strategies we consider: (i) being long an out-of-the-money put option, (ii) being long an out-of-the-money call option, and (iii) being long a risk-reversal (i.e., being long an out-of-the-money put option and short an out-of-the-money call option with symmetric strikes).

[Figure 3 about here.]

A currency option is said to be at-the-money if its strike price is equal to the forward exchange rate. A put (call) option is said to be out-of-the-money if its strike price is below (above) the forward rate—that is, if it takes a large depreciation (appreciation) to make the option worth exercising. The value of an option changes with the value of its underlying asset: the delta of a currency option measures the sensitivity of the option price to changes in the exchange rate. Figure 4 presents the deltas of put options as a function of their strikes. The delta of a put varies between 0 for extremely out-of-the-money options to -1 for extremely in-the-money options. The exercise price of an option can thus be indirectly indirectly characterized by its corresponding delta. A 10 delta (25 delta) put is an option with a delta of -10% (-25%).

[Figure 4 about here.]

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[1]In normal conditions, forward rates satisfy the covered interest rate parity (CIP) condition: forward discounts (i.e., the log differences between forward and spot exchange rates) equal the interest rate differentials between two countries. Akram, Rime, and Sarno (2008) study high-frequency deviations from CIP and conclude that CIP holds at daily and lower frequencies. This relation, however, was violated during the extreme episodes of the financial crisis in the Fall of 2008 (e.g., Baba and Packer, 2009).

1.3   Currency Options

In our data set, options are quoted using their Black and Scholes (1973) implied volatilities for five different deltas. The implied volatility of an option is a convenient normalization of the price of this option as a function of its strike. Our sample comprises monthly deep-out-of-the-money puts (denoted 10 delta puts), out-of-the-money puts (25 delta puts), at-the-money puts and calls, out-of-the money calls (25 delta calls), and deep-out-of-the money calls (10 delta calls) for the January 1996 to August 2014 period. Jorion (1995), Carr and Wu (2007), and Corte, Sarno, and Tsiakas (2011) study the features of currency implied volatilities pre-crisis.

1.4   Smiles

If the underlying risk-neutral distributions of exchange rates were purely log-normal, then implied volatilities would not differ across strike prices. A graph of implied volatilities as a function of their strikes would be flat. Such a flat line is a good description of equity option markets until the crash of 1987. Since 1987, however, equity markets exhibit a different pattern: the price of out-of-the-money options is much higher than the price of at-the-money options. A graph of implied volatilities as a function of strikes thus looks like a "smile."

Currency options exhibit a similar pattern. The dotted lines in Figure 5 are the average implied volatilities for different strikes during the first part of our sample, 1/1996 to 8/2008, for each country, while the lines correspond to the post-crisis sample, from 1/2009 to 8/2014. In both samples, implied volatilities of out-of-the money options tend to be higher than those of at-the-money options. Pre-crisis, out-of-the-money puts and calls roughly exhibit the same implied volatilities (with the exception of the highest-interest rate currencies): in other words, implied volatilities "smile" and those smiles are roughly symmetric over that period. The recent financial crisis introduces a clear change. Post-crisis, currency option smiles are no longer symmetric.

[Figure 5 about here.]

1.5 Risk-Reversals Pre- and Post-Crisis

Risk-reversals offer a simple summary statistic of the asymmetry of the smiles: a high (low) price of an out-of-the-money put option (relative to the price of a call option with symmetric strike) implies a positive (negative) risk-reversal. Until the recent financial crisis, currency risk-reversals were small. Since the crisis, currency option smiles are no longer symmetric and risk-reversals are, in absolute value, an order of magnitude larger than before.

The risk-reversals of the Australian and New Zealand dollars, for example, have notably increased since the beginning of the crisis. For those high interest rate currencies, the risk of large depreciations appears more prevalent than the risk of large appreciations.

In equity markets, a potential interpretation for the high price of out-of-the-money put options and the associated risk-reversal is that equity option prices reflects the possibility of large decreases in stock returns, a potential explanation for the large equity premium. Currency option markets tell a similar story, but in a much more pronounced fashion after 2008 and when comparing low versus high interest rate currencies.

The reason for conditioning on the level of interest rates is simple. Currency markets offer large average excess returns to carry trade investors who go long high interest rate currencies and short low interest rate currencies. In any risk-based view of currency markets, expected carry trade returns compensate investors for bearing the risk of a depreciation (appreciation) of the high (low) interest rate currencies during bad economic periods. In other words, high interest rate currencies are risky, whereas low interest rate currencies are not. But currency markets do not offer significant returns for unconditional investments in any randomly chosen currency. Thus, research on currency returns focuses on conditional investment strategies. In order to study option prices conditional on interest rates, we sort risk-reversals by the level of foreign interest rates and allocate them into three portfolios, which are rebalanced every month. The first portfolio contains risk-reversals from the lowest-interest rate currencies, while the last portfolio contains risk-reversals from the highest-interest rate currencies. Table 1 reports the portfolio average risk-reversals at 10 delta over

different subsample periods.

[Table 1 about here.]

At the portfolio level, the contrast between currency option markets pre- and post-crisis is striking. On average, risk-reversals of high interest rate currencies are equal to 0.6% over the 1996 to 2008 period, while those of low interest rate currencies are equal to -0.7%. During the crisis, the difference in risk-reversals escalates: the risk-reversal of high interest rate currencies reaches 6.4%, while the one for low interest rate currencies declines to -3.9%. After the crisis, on average over the 1/2009 to 08/2014 period, the average risk-reversal of high interest rate currencies is equal to 2.9%, while for low interest rate currencies it is 0.4%. As Figure 6 shows, the difference between the risk-reversals of high and low interest rate currencies is more than twice as large after the recent crisis than before. For high interest rate currencies alone, risk-reversals are six times larger than before the crisis.

[Figure 6 about here.]

The large risk-reversals show that market participants consider the potential for large deprecia- tions of the high interest rate currencies, thus pointing to disaster concerns on currency markets. We now turn to a simple model that establishes the link between currency options and investor compensations for disaster risk exposure.

2   Model

In this section, we describe the pricing kernels, then turn to the implied interest rates, exchange rates, and expected currency returns, as well as the currency option prices in the model.

2.1   Pricing Kernels

The model features two countries: home and foreign. The model is set up and estimated at the monthly frequency, assuming that the parameters that govern the SDF in each country are constant

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over one month. The model parameters, however, are allowed to change non-parametrically the next month. For the sake of clarity, we present the model in two periods. Section 3 shows how to incorporate this building block in a multi-country, multi-period extension. There, a state variable Ω_t describes the state of the world. The parameters of the two-country, two-period model depend on Ω_t. All the results in this section should be understood as returns conditional on Ω_t, but for notational simplicity this dependence is implicit. In particular, all the expectations in this section are conditional on Ω_t.

The SDF for each country incorporates both a traditional log-normal component and a disaster component. SDFs are defined as nominal variables (i.e., expressed in units of local currency) because option data correspond to nominal exchange rates. In the home country, the log of the SDF evolves as:

$$\log M_{t,t+\tau} = -g\tau + \varepsilon\sqrt{\tau} - \frac{1}{2}\operatorname{var}(\varepsilon)\tau$$
$$+ \begin{cases} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J) & \text{if there is a disaster at time } t+\tau \end{cases}.$$

The log of the SDF in the foreign country evolves as:

$$\log M^{\star}_{t,t+\tau} = -g^{\star}\tau + \varepsilon^{\star}\sqrt{\tau} - \frac{1}{2}\operatorname{var}(\varepsilon^{\star})\tau$$
$$+ \begin{cases} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J^{\star}) & \text{if there is a disaster at time } t+\tau \end{cases}.$$

Both SDFs have two components. The first one, $-g\tau + \varepsilon\sqrt{\tau} - \frac{1}{2}\operatorname{var}(\varepsilon)\tau$, is a country-specific Gaussian risk with an arbitrary degree of correlation across countries. Here, g and $g^{\star}$ are constants. The random variables $(\varepsilon, \varepsilon^{\star})$ are jointly normally distributed with mean 0 and are correlated across countries. The second component, $\log(J)$, captures the impact of a disaster on the country's SDF. Disasters are perfectly correlated across the two countries; they are world disasters. The probability of a disaster between t and $t+\tau$ is given by $p\tau$. The Gaussian shocks ε and $\varepsilon^{\star}$ are independent of

the nonnegative random variables J and $J^\star$, which measure the magnitudes of the disaster event. All these variables are independent of the realization of the disaster event. At the start of each month, the magnitudes of disasters are unknown. To model their randomness in a parsimonious way, we assume that the impacts of a global disaster on the home and foreign SDFs are:

$$J(\eta) = \bar{J} \cdot (1 + \eta\sigma_J), \, J^*(\eta^*) = \bar{J}^* \cdot (1 + \eta^*\sigma_{J^*}),$$

where η and η^* are i.i.d. Bernoulli variables equal to 1 and -1 with equal probability and σ_J and σ_J^* govern the amount of uncertainty on disaster sizes. This uncertainty implies that the foreign currency may appreciate or depreciate vis-à-vis the home currency when a disaster happens.

The term "disaster" can have several interpretations. One, championed by Rietz (1988) and Barro (2006), is that of a macroeconomic drop in aggregate consumption, perhaps due to a war or a major economic crisis that affects many countries. Another interpretation is that of a financial stress or crisis affecting participants in world financial markets, perhaps via a drastic liquidity shortage or a violent drop in asset valuations. Both interpretations have merit, and we do not need to take a stand on the precise nature of a disaster. In our setting, a disaster is a large increase in the SDFs. The laws of motion of the domestic and foreign SDFs are enough to compute all relevant asset prices, starting with interest rates, exchange rates, and expected currency returns.

2.2 Interest Rates, Exchange Rates, and Expected Currency Excess Returns

Let us first define exchange rates. As in Bekaert (1996) and Bansal (1997), the change in the (nominal) exchange rate is given by the ratio of the SDFs:

$$\frac{S_{t+\tau}}{S_t} = \frac{M^\star_{t,t+\tau}}{M_{t,t+\tau}}, \tag{1}$$

where S is measured in home currency per foreign currency. An increase in S represents an appreciation of the foreign currency (we use the same sign convention as in the data analysis). Just like

the exchange rate allows us to convert the home price of a good into foreign currency, it also allows us to convert the home currency SDF into the foreign currency SDF.

It might seem counterintuitive that when the foreign SDF increases more than the home SDF, the foreign currency appreciates. However, this robust implication of finance theory is a simple matter of accounting (and is not specific to disaster models) and can be thought as a version of the Law of One Price. The marginal investor can assess a given return either in home ($R_{t,t+\tau}$) or foreign currency ($R^*_{t,t+\tau} = R_{t,t+\tau} \frac{S_t}{S_{t+\tau}}$). The unit of account is simply a veil and has no impact on intrinsic valuation. The home currency SDF, $M_{t,t+\tau}$, and foreign currency SDF, $M^*_{t,t+\tau}$, encode the valuation of returns in home and foreign currency by the same marginal investor. This requires that $E[M_{t,t+\tau} R_{t,t+\tau}] = E[M^*_{t,t+\tau} R^*_{t,t+\tau}]$, for all equilibrium home currency returns, $R_{t,t+\tau}$. This immediately implies Equation (1).[2]

Let us turn now to interest rates; likewise, they are pinned down by the two SDFs. The home interest rate r is determined by the Euler equation $1 = E[M_{t,t+\tau} e^{r\tau}]$:

$$r = g - \log\left(1 + p\tau E[J - 1]\right)/\tau. \tag{2}$$

A similar expression determines the foreign interest rate. Currency carry trades then correspond to the following investment strategy: at date t, the investor borrows one unit of the home currency at rate r and invests the proceeds in the foreign currency at rate r^*. At the end of the trade, at date $t + \tau$, the investor converts the proceeds back into the home currency. In units of the home currency, the payoff to the currency carry trade is:

$$X_{t,t+\tau} = e^{r^*\tau} \frac{S_{t+\tau}}{S_t} - e^{r\tau}.$$

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[2]An alternative derivation of Equation (1) starts from the Euler equations $E[M^*_{t,t+\tau} R^*_{t,t+\tau}] = 1$ and $E[M_{t,t+\tau} R^*_{t,t+\tau} \frac{S_{t+\tau}}{S_t}] = 1$ of two different investors, home and foreign. If financial markets are complete, then the SDF is unique, and the exchange rate is defined in terms of SDFs. Note that real exchange rates are time-varying even when financial markets are complete, as long as some frictions in the goods markets prevent perfect risk sharing across countries. An example of such a friction often used in the literature is the assumption that some goods are not traded.

In the limit of small time intervals, interest rates and expected currency excess returns take a very simple form, presented in Proposition 1.

Proposition 1. *In the limit of small time intervals $\tau \to 0$, the interest rate r in the home country is:*

$$r = g - pE\left[J - 1\right].\tag{3}$$

Carry trade expected returns (conditional on no disasters) are given by:

$$X^e = \pi^D + \pi^G,\tag{4}$$

where:

$$\pi^D = pE\left[J - J^\star\right],\tag{5}$$

$$\pi^G = cov(\epsilon, \epsilon - \epsilon^*).\tag{6}$$

The interest rate has two components: the drift of the SDF and the disaster element. A foreign country whose currency tends to depreciate in times of disasters against the home currency (i.e., $E\left[J^*\right] < E\left[J\right]$) exhibits an interest rate that is above the home interest rate.[3]

The currency excess return also has two components. The first term in Equation (4) is the investor compensation associated with disaster risk:

$$\pi^D \equiv pE\left[J - J^\star\right].$$

If $E\left[J - J^\star\right] > 0$, the expected return due to disaster risk is positive because the foreign currency tends to depreciate when disasters occur. The second term in Equation (4) is the compensation

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[3]Farhi and Gabaix (2013) provide a detailed micro-foundation for the variables J and J^* that has two implications: (i) the more severe the world disasters (so that the world consumption of tradable goods falls more), the higher the values of J and J^*; (ii) if the foreign country fares worse than the home country in times of disasters (which implies that its currency depreciates when disasters occur), then J^* is less than J.

associated with "Gaussian risk" à la Backus, Foresi, and Telmer (2001):[4]

$$\pi^G \equiv \text{cov}\left(\varepsilon, \varepsilon - \varepsilon^\star\right).$$

This is the covariance between the home SDF and the bilateral exchange rate, $S_{t+\tau}/S_t$. If a foreign currency tends to depreciate during bad economic periods, investors expect to be compensated by a positive premium. In our model, the expected return of the carry trade compensates for the exposure to these two sources of risk.

The disaster risk exposure (π^D) corresponds to the part of the expected excess return due to disaster risk in a sample without disasters. The disaster risk premium is the disaster risk exposure, minus the expected loss during a disaster:

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\begin{aligned}
\text{Disaster Risk Premium} \quad &= \quad (1-p)E[X_{t,t+\tau}|No\ Disaster] + pE[X_{t,t+\tau}|Disaster] \quad\quad (7)\\
&\simeq \quad \pi^D + pE[X_{t,t+\tau}|Disaster].
\end{aligned}
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Consistent with the disaster risk literature, we focus much of the analysis on π^D, and will come back to the risk premium later. The discrepancy between the two will prove to be moderate – about $2/3$ of π^D is a risk-premium, rather than an expected loss. Moreover, for currency prices, in many models in which the exchange rate is the expected present value of future fundamentals, it is the disaster exposure that matters to determine the value of a currency, rather than the disaster risk premium per se. For example, in Farhi and Gabaix (2013) the relative exchange rate between two countries is driven by the difference in resiliences of the two countries, which is exactly π^D in our notations (in the limit of small time intervals and resiliences).[5] Finally, suppose that one takes the

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[4]Backus, Foresi, and Telmer (2001) show that, if markets are complete and SDFs are log normal, then expected log currency excess returns are equal to $E[\log R^e] = 1/2 Var(\log M) - 1/2 Var(\log M^\star)$. However, the focus here is on the log of expected currency excess returns, but the two expressions are naturally consistent.

[5]The correspondence is as follows: our $p(J_i - 1)$ is equal to the resilience $H_i = p(B^{-\gamma}F_i - 1)$ in Farhi and Gabaix's (2013) notations, where $B^{-\gamma}$ is the growth of marginal utility of world consumption of the tradable good and F_i is the recovery rate of country i's productivity, both in a disasters. So, our π_D is the difference in the resiliences H_i of the two countries.

view that the Fall of 2008 is an increase of the disaster probability, not a full-blown disaster. Then, the whole time series of monthly exchange rates among developed countries (roughly, the 1970s until now) does not contain any disaster. Hence, to compare theory to data, one needs to specify the predictions of a theory for a sample that does not ex post contain a disaster, although disasters were feared all along in that sample.

2.3   Option Prices

We turn now to option prices in the model. $P_{t,t+\tau}$ is the home currency price of a put with strike K bought at date t and maturing at date $t+\tau$, thus yielding $(K - S_{t+\tau}/S_t)^+$ in the home currency, with the usual notation of $y^+ \equiv \max(0, y)$. The home (here U.S.) investor starts with one U.S. dollar, i.e., $1/S_t$ units of foreign currency. If the exchange rate at the end of the contract is lower than the strike ($KS_t > S_{t+\tau}$, where K is measured in units of foreign currency), then the put contract pays off the difference between the strike and the spot rate, $S_{t+\tau}$, for each unit of foreign currency invested; the payoff per U.S. dollar is thus $(K - S_{t+\tau}/S_t)^+$. Likewise, $C_{t,t+\tau}$ is the home currency price of a call yielding $(S_{t+\tau}/S_t - K)^+$ in the home currency. Put and call prices in the model can be expressed using the Black and Scholes (1973) formula, even though the model features non-Gaussian shocks.

2.3.1   Option prices in a Gaussian world

The Black and Scholes (1973) formula, developed originally in the context of stock markets, was adapted to a foreign exchange setting by Garman and Kohlhagen (1983). Let $V_{BS}^P(S, \kappa, \sigma, r, r^\star, \tau)$ and $V_{BS}^C(S, \kappa, \sigma, r, r^\star, \tau)$ denote the Black and Scholes (1973) prices for a put and a call, respectively, when the spot exchange rate is S, the strike is κ, the exchange rate volatility is σ, the home interest rate is r, the foreign interest rate is $r^\star$, and the time to maturity is τ. The prices of a call and a

put are given by:

$$
\begin{aligned}
V_{BS}^C(S, \kappa, \sigma, r, r^\star, \tau) &= Se^{-r^\star \tau}\mathbb{N}(d_1) - \kappa e^{-r\tau}\mathbb{N}(d_2), \\
V_{BS}^P(S, \kappa, \sigma, r, r^\star, \tau) &= \kappa e^{-r\tau}\mathbb{N}(-d_2) - Se^{-r^\star \tau}\mathbb{N}(-d_1), \\
d_1 &= \frac{\log(S/\kappa) + (r - r^\star + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \\
d_2 &= d_1 - \sigma\sqrt{\tau},
\end{aligned}
$$

where $\mathbb{N}$ is the Gaussian cumulative distribution function. The Black and Scholes (1973) and Garman and Kohlhagen (1983) formulas have a simple scaling property with respect to the time to maturity τ and the interest rates r and $r^\star$:

$$
V_{BS}^P(S, \kappa, \sigma, r, r^\star, \tau) = V_{BS}^P(Se^{-r^\star \tau}, \kappa e^{-r\tau}, \sigma\sqrt{\tau}, 0, 0, 1).
$$

For notational convenience, the arguments 0 and 1 are omitted and the value of a generic put is simply $V_{BS}^P(S, \kappa, \sigma) = V_{BS}^P(S, \kappa, \sigma, 0, 0, 1)$.

2.3.2   Option prices in the model

Let us turn now to the price of a put in the model. The price of a call is derived similarly. We define $\tilde{J} = \frac{p\bar{J}}{1-p\tau}$ and $\tilde{J}^* = \frac{p\bar{J}^*}{1-p\tau}$ and use them in Proposition 2 for mathematical convenience. Economically, however, $\tilde{J}$ and $\tilde{J}^*$ are empirically close to $p\bar{J}$ and $p\bar{J}^*$ at the one-month horizon for any reasonable disaster probability.

Proposition 2. *In our model, the put option price is given by:*

$$
\begin{aligned}
P_{t,t+\tau}(K, \tilde{J}, \tilde{J}^*, \sigma_J, \sigma_{J^*}, \sigma_h) = E\Big[&P_{t,t+\tau}^{ND}\Big(K, \tilde{J}\cdot(1+\eta\sigma_J), \tilde{J}^*(1+\eta^*\sigma_{J^*}), \sigma_h\Big) \\
&+ P_{t,t+\tau}^{D}\Big(K, \tilde{J}\cdot(1+\eta\sigma_J), \tilde{J}^*(1+\eta^*\sigma_{J^*}), \sigma_h\Big)\Big]
\end{aligned}
$$

where:

$$P_{t,t+\tau}^{ND}(K, \tilde{J}, \tilde{J}*, \sigma_h) = V_{BS}^{P}\left(\frac{e^{-r^*\tau}}{1 + \tilde{J}^*\tau}, K\frac{e^{-r\tau}}{1 + \tilde{J}\tau}, \sigma_h\sqrt{\tau}\right),$$

$$P_{t,t+\tau}^{D}(K, \tilde{J}, \tilde{J}*, \sigma_h) = \tau V_{BS}^{P}\left(\frac{e^{-r^*\tau}\tilde{J}*}{1 + \tilde{J}^*\tau}, K\frac{e^{-r\tau}\tilde{J}}{1 + \tilde{J}\tau}, \sigma_h\sqrt{\tau}\right),$$

and the strike is K, the time to maturity is τ, the home interest rate is r, the foreign interest rate is r^*, the volatility of the Gaussian part of exchange rates is $\sigma_h = \sqrt{var(\varepsilon - \varepsilon^*)}$, and the expectation is taken over η, η^*, which are i.i.d. Bernoulli variables with values in $\{-1, 1\}$.

The closed-form expression implies a natural estimation procedure, minimizing the distance between actual and model-implied option prices.

2.3.3 Estimation procedure

For each country indexed by i, each quoted strike j and at each date t, we consider the difference between the quoted put price, P_{ij}, and its model counterpart, $P(K_{ij}, \tilde{J}, \tilde{J}^*_i, \sigma_J, \sigma_{J^*i}, \sigma_{hi})$. Put-call parity implies that call prices reflect the same information as put prices. The model parameters ($\tilde{J}$, $\tilde{J}^*$, σ_J, σ_{J^*}, σ_h) are obtained, at each date t and for each foreign country, by minimizing the sum of squared price differences across countries and strikes:

$$\min_{\tilde{J},\tilde{J}^*_i,\sigma_J,\sigma_{J^*i},\sigma_{hi}} \sum_{i=1}^{9}\sum_{j=1}^{5}\left[P_{ij} - P(K_{ij}, \tilde{J}, \tilde{J}^*_i, \sigma_J, \sigma_{J^*i}, \sigma_{hi})\right]^2,$$

where the expression for the put in the model is given in Proposition 2. The relatively larger prices of at-the-money and close-to-the-money options imply that the minimization algorithm focuses on them. Our estimation is thus conservative, focusing on the most liquid and less disaster-prone currency options. Larger weights on the 10 delta options, for example, would likely increase the share of disaster risk.

Since the model parameters move freely across time periods, minimizations are independent across time, but they are not independent across currencies, because all exchange rates depend on the characteristics of the U.S. SDF. The estimation of the model is therefore implemented jointly for all currency pairs, date by date. At each date, each currency is characterized by its disaster risk exposure, $\tilde{J}^*$ and σ_{J^*}, as well as its Gaussian volatility, σ_h. The U.S. exposure to disaster risk is governed by $\tilde{J}$ and σ_J. The 9 currency pairs defined with respect to the U.S. dollar are thus characterized by 29 ($3 \times 9 + 2$) parameters at each date. The estimation uses 45 option prices (5 strikes for each currency pair) and finds the global minimum over a grid of initial conditions.

2.4 Key Assumptions

Before turning to the data to implement the estimation procedure above, we first assess the validity of the experiment. The model is extremely tractable; indeed, it yields closed-form solutions for a number of key moments. The model is also very flexible; it allows the realized and expected volatilities of exchange rates to be time-varying, in line with previous findings on currency markets (e.g., Diebold and Nerlove, 1989). The volatilities are held constant over one month and then move non-parametrically from one month to the next.

The tractability and flexibility rely on two key assumptions: the shocks ϵ and ϵ^* are (i) jointly normal, and (ii) independent from J, J^*, and the realization of the disaster. Excluding the Fall of 2008, the difference $\epsilon^* - \epsilon$ appears conditionally normally distributed (as shown by a Jarque-Bera test), once one controls for the time-varying volatility of exchange rates. Yet, in the model, we presume not only that the difference $\epsilon^* - \epsilon$ is normal but also that the shocks ϵ and ϵ^* are both normal and independent of the realization of disasters. This log-normality and independence assumption on pricing kernels cannot be tested with exchange rates alone, but is common across macroeconomic models of exchange rates. The empirical experiment that follows is thus run under the assumption that SDF shocks at the monthly frequency are conditionally Gaussian when no disaster occurs.

3   Estimation of Disaster Risk Exposure

This section reports estimates of currency excess returns and compensations for disaster risk expo-
sure using option prices. We proceed in several steps. First, in Section 3.1, we sort currencies into
portfolios based on their interest rates. We then report a number of characteristics of these port-
folios k: the average expected appreciation of the currencies in each portfolio, the average interest
rate differential with the U.S. in each portfolio, and most importantly, the average dollar excess
return $\overline{X_k}$ of strategy that borrows in U.S. dollars and invests in each portfolio.[6] Then in Section
3.2, we use the estimation described in Section 2.3.3 to compute the average disaster exposure $\overline{\pi_k^D}$
for each portfolio, and the disaster share $\overline{\pi_k^D}/\overline{X_k}$. We also compute an estimate of the disaster risk
premium.[7]

3.1   Currency Portfolios

We build portfolios of currency excess returns in order to focus on the sources of aggregate risk and
to average out idiosyncratic variations. At the portfolio level, high interest rate currencies deliver
average currency excess returns that are significantly different from zero; they capture expected
excess returns from currency markets. We first describe the portfolio sorts and the sample period
and then turn to the portfolio characteristics.

3.1.1   Portfolios Sorts

For each individual currency, the corresponding excess return is built from the perspective of a U.S.
investor. The first portfolio contains the lowest interest rate currencies while the last portfolio
contains the highest interest rate currencies. Inside each portfolio, currencies are equally-weighted.

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[6]Note that the average dollar excess return $\overline{X_k}$ is computed without any reference of the estimation procedure
described in Section 2.3.3. In particular, it is different from the variables $\overline{X_k^e}$ which uses the estimated model to
compute an average expected excess return conditional on no disasters.

[7]To compute an estimate of disaster risk premia, we need to estimate more parameters than what the procedure in
Section 2.3.3 allows us to recover. Indeed, we need an estimate of the disaster probability and the expected loss during
a disaster, for which we use an estimate of the disaster probability from Barro and Ursua (2008) and the cumulative
return in 2008 in the portfolio of high interest rate currencies.

The connection with the theory developed in Section 2 is as follows. The different countries are indexed by $i \in I$. A state variable Ω_t describes the state of the world at date t. This state variable follows an arbitrary stationary stochastic process. All the parameters of the model are arbitrary functions of Ω_t. Correspondingly, all the computed variables r_i, X_i^e, π_i^D, and π_i^G depend on Ω_t. Underlying our three portfolios are three state-dependent sets: $I_1(\Omega_t)$, $I_2(\Omega_t)$, and $I_3(\Omega_t)$. Forming portfolios is a way to compute moments conditional on the three sets: I_1, I_2, and I_3. For instance, the average disaster exposure in portfolio k is simply the average of the disaster exposure over the countries in the portfolio:

$$\overline{\pi_k^D} = E\left[\frac{\sum_{i \in I_k(\Omega_t)} \pi_i^D(\Omega_t)}{\#I_k(\Omega_t)}\right],$$

where I_k denotes the set of currencies in portfolio k and $\#I_k(\Omega_t)$ denotes their number.

3.1.2 Sample Period

In the sample period, Fall 2008 appears as the unique potential example of disasters and thus deserves special attention. An investor borrowing in Japanese yen and lending in New Zealand dollars would have incurred a loss of almost 30% in October 2008, and a total loss of close to 40% in the Fall of 2008. In a diversified portfolio of high and low interest rate currencies, the average return of the carry trade strategy is -4.5% in the Fall of 2008, for a cumulative decline from September to December 2008 of 13.6%. This is a large drop, as the standard deviation of monthly returns over the whole sample is just 2%. Almost all of the 13.6% decline is due to losses on high interest rate currencies, which depreciated sharply. The large changes in exchange rates triggered the exercise of currency options. For example, in our sample, the share of 10 delta put options exercised reaches an all-time high in the Fall of 2008.

These very low returns on currency markets occurred during a poor economic period for U.S. and world investors (see Lustig and Verdelhan, 2007, 2011). During Fall 2008, the U.S. stock market declined by 33% in terms of the MSCI index. The closest event to this very strong decline in equity and currency returns is the 1987 stock market crash: from September to November 1987,

the U.S. stock market lost 32.6%. Standard risk measures beyond those from equity markets point in the same direction. Very low currency excess returns (four standard deviations below their means) happened exactly when volatilities in equity and bond markets and credit spreads were high (four standard deviations above their means). These market-based indices offer real-time measures of risk that complement the approach based on marginal utilities and real consumption growth rates. U.S. national account statistics point toward an annualized decrease of 4.3% in real personal consumption expenditures in the fourth quarter of 2008, following an annualized decrease of 3.8% in the third quarter. These shocks represent declines of more than three standard deviations in the mean consumption growth rate.

There are two interpretations of Fall 2008, as a disaster, or as a temporary sharp increase in the probability of disaster.

First, suppose that Fall 2008 is viewed as an example of disasters in our sample. This view is consistent with our model, which implies that, as long as a currency crash does not occur in the sample, *conditional* monthly changes in exchange rate are conditionally normally distributed. This is indeed the case if the Fall of 2008 is excluded from the sample. To take into account exchange rate heteroscedasticity, a GARCH (1,1) model is estimated for each currency and then normality tests are run on exchange rate changes normalized by their volatility. After the GARCH (1,1) correction, all countries exhibit conditionally Gaussian exchange rates in the sample. Since our decomposition of expected currency excess returns is valid in samples without disasters, we report results on samples that exclude Fall 2008 when that decomposition is used.

Second, suppose that Fall 2008 is viewed as an (temporary) increase in the probability of disasters, not the realization of one particular disaster. For robustness checks, we also report average estimates of the compensations for disaster risk exposure on samples that include the Fall of 2008. In that view, conditional changes in exchange rates are normally distributed in the Fall of 2008 as in the rest of the sample. The results of conditional normality tests depend naturally on the information set and the conditioning variables used, and are thus subject to discussion. The main findings in this paper do not depend on such discussion.

3.1.3 Portfolio Characteristics.

Let us turn now to the characteristics of the portfolios. Table 2 reports average changes in exchange rates, interest rates, risk-reversals at 10 and 25 delta, as well as average currency excess returns over the period from January 1996 to August 2014. These numbers are simple averages of the corresponding numbers over the currencies in this given portfolio over time. They make no use of the estimates produced by the estimation procedure outlined in Section 2.3.3. In the Online Appendix, we show similar results when we exclude Fall 2008 from the sample.

[Table 2 about here.]

Average currency excess returns increase monotonically from the first to the last portfolio. This is not a surprise: we know from the empirical literature on the uncovered interest rate parity that high interest rate currencies tend to appreciate on average. As a result, investors in these currencies gain both the interest rate differential and the foreign exchange rate appreciation. Excess returns on high interest rate currencies are 4.3% (5.4%) on average including (excluding) the Fall of 2008 and are more than two standard errors away from zero. The currency excess returns imply a 0.4 (0.6) Sharpe ratio, which is higher than the Sharpe ratio on the U.S. equity market over the same period.

If disaster risk is an important determinant of cross-country variations in interest rates, then a portfolio formed by selecting countries with high interest rates will, on average, select countries that feature a large risk of currency depreciation. We will come back to this point after estimating each country's disaster risk exposure, but risk-reversals give a preliminary hint. Intuitively, as already noted in Section 1, higher probabilities of depreciation for the foreign currency should show up in higher levels of risk-reversals. Thus, if disaster risk matters for the cross-country differences in interest rates, high interest rate countries should exhibit high risk-reversals; Table 1 already shows that for risk-reversals at 10 delta. Table 2 reports similar evidence for risk-reversals at 25 delta. Risk-reversals at 10 and 25 delta increase monotonically across portfolios. Similar results are obtained when the Fall of 2008 is included in the sample. The results confirm and extend the

previous findings of Carr and Wu (2007), who report a high contemporaneous correlation between currency excess returns and risk reversals for the yen and the British pound against the U.S. dollar. Note that the risk reversals at 10 delta are more expensive than those at 25 delta. This is again consistent with a risk of depreciation for high interest rate currencies.

Currency markets thus exhibit large average excess returns that seem potentially linked to disaster risk. We now turn to the estimation of the market's compensation for bearing such risk.

3.2 Disaster Risk

In this section, we use the closed-form expressions of option prices presented in Section 2.3 to estimate a time series of disaster risk exposure and an average disaster risk premium.

3.2.1 Average Disaster Risk Exposure

Estimates using the procedure outlined in Section 2.3.3 are obtained for each country and each date. For the sake of clarity, we then aggregate the results at the portfolio level and focus on the portfolio of high interest rate currencies, which exhibits significant average excess returns. Time series of the country-level estimates are reported in the Online Appendix. Table 3 reports estimates of average disaster risk exposure over different time-windows. In the full sample, the compensation for disaster risk exposure is significantly different from zero: it is 2.3% on average, accounting for 53.5% of the 4.3% of total currency excess return (Panel I). Excluding the Fall of 2008, the disaster risk exposure is 2.2%, which is 40% (Panel II) of the total average currency excess return. Over the pre-crisis period, the role of disaster risk is statistically significant, but economically small: the compensation for disaster risk is less than 0.5%, accounting for less than 13% of total currency excess return (Panel III). The 1996 to 2007 period thus offers only limited support to the disaster risk model. Over the post-crisis period, however, disaster risk appears as a major concern of market participants, as it accounts for more than half of the total currency risk compensation (Panel IV). Disaster risk is thus priced in currency markets and requires a sizable compensation, particularly

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over the recent period.

[Table 3 about here.]

3.2.2 Time Series of Disaster Risk Exposure

Figure 7 presents the time series estimates of the compensation for disaster risk exposure ($\pi^D \equiv pE[J - J^\star]$, top panel) and of the volatility parameter (σ_h, bottom panel) for the high interest rate currencies. Consistent with the averages presented in Table 3, the expected disaster risk exposure is low over the 1996 to 2007 sample, but it increases markedly with the recent financial crisis and has remained at high levels since then. This increase in disaster risk exposure is intuitive; it mirrors the increase in risk-reversals noted in the previous section. At the country level, the correlations between risk-reversals and estimates of disaster risk exposure vary between 0.70 and 0.93 depending on the country. The Fall of 2008 is also characterized by a large increase in expected exchange rate volatility: yet, the volatility decreased after the crisis, while the compensation for disaster risk has not. The estimation also reveals that the Asian crisis of 1998 did not affect the price of disaster risk for the developed countries in our sample. In this perspective, the Asian crisis is not interpreted as a world disaster by currency option markets, but merely as a limited increase in expected exchange rate volatility.

[Figure 7 about here.]

The model and its associated estimation thus deliver the expected goal: a simple, time-varying, real-time estimate of the expected exposure to global disaster risk. This is the key contribution of the paper.

3.2.3 Disaster Risk Premium

The empirical analysis above allowed us to estimate the disaster exposure, $\pi^D \equiv pE[J - J^\star]$. In order to estimate a disaster *risk premium* as defined in Equation (7), one needs to estimate the

expected loss during a disaster and the disaster's probability. Our simple model and estimation procedure do not allow for separate estimations of disaster probabilities and disaster sizes. A back-of-the-envelop estimate of the disaster risk premium, however, can be obtained using 2008 as an example of a disaster and estimates of the disaster probabilities in the literature.

The cumulative excess return in 2008 in the portfolio of high interest currencies is -19.4%. Barro and Ursua (2008) estimate the disaster probability at 3.63% per annum. Using those estimates, the expected currency carry trade loss is then equal to -0.7% ($3.63\% \times (-19.4\%)$). There is a substantial uncertainty about this number. For instance, if 2008 is simply an increase in the disaster probability, then the expected disaster loss could be higher—this would increase our estimate. Barro (2006) estimates a disaster probability of 1.7% per year—taking this number would lower our estimate of expected losses. Assuming an expected currency excess return of -0.7% in times of disasters leads to a risk premium of 1.6% ($2.3\% - 0.7\%$), which corresponds to 37.2% of the average carry trade excess return (4.3%) in our sample.

Note that the remaining "Gaussian risk" may come from disaster risk itself.[8] In disaster models with Epstein-Zin preferences, variations in the aggregate disaster probability creates Gaussian risk (e.g., Gabaix, 2012, Du, 2013, Wachter, 2013), but this Gaussian risk itself, intrinsically, stems from the time-varying fears of disasters. Under that interpretation, the share of total risk due to disasters would be higher than one-third. We focus on this conservative estimate because Gaussian risk could also come from very different models (e.g., models featuring habits or long run risks).

3.3 Robustness

In this section, we assess the robustness of our results to four empirical issues: the relative weights on options, the mis-measurement due to transaction costs, model mis-specifications, and the monthly frequency of the data.

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[8]We thank Jessica Wachter for pointing this out.

3.3.1 Relative Option Weights

Our benchmark estimate implicitly puts more weight on the at-the-money and 25-delta options than on the 10-delta options because of their different price magnitudes. As a robustness check, we estimate all the model parameters by minimizing the percentage gap between the model and actual option prices, therefore neutralizing any scale effect. As expected, this estimation puts more weight on the out-of-the-money 10 delta options and the disaster risk exposure increases to 2.7% over the whole sample (excluding the Fall of 2008). As a result, the share of currency excess return explained by disaster risk increases from 40% to 49.6%. Our benchmark estimate therefore appears conservative; estimates that rely on relatively less traded out-of-the money options lead to even higher disaster risk exposure.

3.3.2 Transaction Costs

Our benchmark estimates of the compensation for disaster risk exposure do not take into account bid-ask spreads on currency markets. Transaction costs on forward and spot contracts reduce excess returns, while transaction costs on currency options increase insurance costs. We propose a preliminary estimation of their impact, constrained by data availability.

The dataset includes bid and ask quotes on the spot and the forward exchange rates for the entire sample. Unfortunately, bid and ask quotes on currency options are only available after 9/2004 and for a limited set of countries (Australia, Canada, Euro area, Japan, Switzerland, and U.K.) on Bloomberg. The bid-ask spreads are expressed in units of implied volatilities for each strike. On this limited sample, bid-ask spreads are clearly larger out-of-the-money than at-the-money. Bid-ask spreads appear stable pre-crisis, over the 9/2004 to 3/2007 period. To extend the bid and ask series to the earlier part of our sample (1/1996–8/2004), we thus use the cross-country average bid-ask spread measured on the pre-crisis period for each strike. To extend the series to Norway, New Zealand, and Sweden after 2004, the cross-country average bid-ask spread at each point in time and for each strike is used. As a result, bid-ask spreads widen when implied volatilities increase.

The implied volatilities spreads are converted into bid-ask prices in order to re-estimate Gaussian and disaster risk exposure.

After bid-ask spreads, average currency excess returns over the whole sample (excluding the Fall of 2008) on the high interest rate portfolio decrease from 5.4% to 4.5%, while the disaster premium decreases from 2.2% to 1.4%. As a result, the share of currency excess return explained by disaster risk decreases from 40% to 31.3%. Overall, the results appear robust to the introduction of transaction costs and, again, our benchmark results appear conservative.

Note, however, that the estimation above does not rule out more serious illiquidity issues. It is possible to imagine that the J.P.Morgan market maker simply gives indicative prices by using the Black and Scholes (1973) formula (which generates a low option price), but there is little trading of out-of-the-money options. If someone wanted to aggressively buy these options, then she would end up moving prices against herself and paying higher prices. If this is the case, the potential trading prices are higher than the indicative prices in our data, and disaster risk is thus under-estimated.

3.3.3 Model Misspecification

The model may be misspecified, and not fully capture the richness of option dynamics. It ignores any potential market segmentation between currency markets and other asset markets, and does not account for the full term structure of interest rates. One way to address these concerns would be to extend the model but at the cost of losing tractability and focus. A natural extension would be the introduction of small disasters. In such a specification, out-of-the-money options offer no protection against small disasters and would therefore be cheaper than at-the-money options. We choose instead to maintain the parsimony of the model and show that its focus on large disasters is consistent with the average cross-country differences in interest rates over the sample and the changes in exchange rates during the financial crisis, while producing small pricing errors.

First, as already noted in the introduction and shown in Figure 1, high interest rate countries are characterized by large disaster risk exposure on average. The finding is not mechanical because the model allows for a free drift parameter that could potentially account for the cross-country

differences in interest rates. The finding is consistent with Brunnermeier, Nagel, and Pedersen (2008), who show that high interest rate countries tend to exhibit high risk-reversals in the pre-crisis sample. In the post-crisis sample, the link is much stronger, as Section 1 shows. Our estimation procedure extracts the disaster risk exposure from option prices and highlights the link between interest rates and the risk of large currency movements.

Second, the core mechanism of the model is the risk of large currency fluctuations in times of global disasters. If one interprets the Fall of 2008 as an example of such global disaster, the model's implications are clearly borne out in the data. As Figure 2 shows, realized changes in exchange rates are consistent with estimates of disaster risk exposure from currency options. This result is not mechanical either as the estimation of disaster risk does not use changes in exchange rates. The finding is consistent with the rest of the paper: in the model, high interest rate currencies bear the risk of large depreciations in times of disaster, and thus offer high expected excess returns due to large disaster risk exposure. In the data, high interest rate currencies depreciated sharply in the Fall of 2008, while low interest rate currencies appreciated. Again, the estimation procedure extracts the disaster risk exposure from option prices, and it appears consistent with the behavior of exchange rates during the crisis.

Finally, the model fits the data very well: the average option pricing errors appear small compared to the bid-ask spreads. Pricing errors are computed as the absolute difference in implied volatility between the model and the data. Table 4 reports, for each strike, the square root of the mean squared pricing errors and the square root of the mean squared of the bid-ask spreads obtained for the portfolio of high interest rate currencies.

[Table 4 about here.]

All of the average pricing errors (for all strikes and samples) are smaller than the bid-ask spreads. The empirical and cumulative distributions of the time series of the absolute pricing errors, which we show in the Online Appendix, confirm this result. The estimation delivers small pricing errors compared to the uncertainty in the option prices as measured by their bid-ask spreads. The small

pricing errors indicate that the model captures well the dynamics of the option prices.

3.3.4 Estimation Frequency

Our model is written and estimated at the monthly frequency and we focus on a simple carry trade strategy implemented through hypothetical portfolios. The model thus abstracts from higher frequency portfolio choices and more sophisticated investments. One could argue that sophisticated investors would not be sensitive to changes that take place over one month; however, data on hedge fund returns suggest otherwise.

The Morningstar CISDM database contains 158 hedge funds following a global macro strategy, including both active and defunct funds (135 funds were active in August 2008, and 131 in September 2008). The oldest hedge fund in the sample began operation in 1986, but the majority of the funds became active in the 2000s. Since actual hedge fund trades are not observable, we focus on funds whose returns load on the carry trade factor of Lustig, Roussanov, and Verdelhan (2011) by estimating the following two-factor model:

$$R_{i,t} = \alpha_i + \beta_i HML_t^{FX} + \beta_i^w RW_t + \varepsilon_{i,t},$$

where $R_{i,t}$ is the return of hedge fund i at date t, HML_t^{FX} is the return of high interest rate currencies minus the return on low interest rate currencies, and RW_t is the world stock market return measured by the Dow Jones Global Index. The carry trade betas (β_i) and world market betas (β_i^w) are estimated on the 24-month period that ends in August 2008. Similar results are obtained with estimation windows of 36 and 48 months. The carry trade betas strongly predict currency returns in September 2008, even after controlling for world market betas:

$$R_i^{9/2008} = \gamma + \delta\beta_i + \delta^w\beta_i^w + \eta_i.$$

The R^2 of this regression is 47% (vs. 10% when only the world markets betas are included) and both

slope coefficients are highly significant. All hedge funds versed in carry-trade strategies apparently did not get a chance to exit before the carry trade returns collapsed and some endured large related losses in September 2008. The mean return among the hedge funds with the largest carry trade betas (fifth quintile) is -5.1%. Subtracting the exposure to world stock markets ($\delta^W \beta_i^w$), the mean return is still -3.6%. It is low compared to the mean return over the previous year (1.0%) and compared to the standard deviation of around 0.8% of the portfolio return over the previous three years. The decrease of -3.6% on a portfolio of hedge funds thus represents a decrease of more than four standard deviations. Moreover, the averages per quintile hide large losses for some hedge funds, some reaching a minimum of -24% in September 2008. The strong predictive power of the carry trade betas indicates that carry risk played a large role in the low returns experienced by hedged funds in the Fall of 2008. Although our model ignores higher frequency variation, it captures a first-order economic effect of disasters.

Our estimation thus appears robust to several concerns. A final concern lies in the existence of counterpart risk, in the case of options without large enough margins. The counterparty risk issue relies on the possibility that the seller of a put might actually default during a disaster. Put premia take that risk into account and are lower than in the model. We expand on this question in the next section.

4   Disaster Risk Across Markets

We use the estimated time-series of disaster risk exposure in order to test some key model implications and uncover new contemporaneous links across asset markets. We first focus on the link between disaster risk exposures and either interest rates or exchange rates, and finally turn to the link between equity risk and disaster risk.

4.1 Disaster Risk and Interest Rates

Our model predicts that, in the limit of small time intervals, interest rates in country j can be expressed as a simple function of disaster risk exposure in that country (cf Proposition 1). Figure 1 tests this implication in the cross-section of average interest rates. In this section, we focus on the time-series. For each country j, we run the following contemporaneous regression of short-term interest rates on disaster risk exposures:

$$r_{j,t} = \alpha_j + \beta \widehat{pJ^*_{j,t}} + \epsilon_{j,t},$$

where $\widehat{pJ^*_{j,t}}$ is estimated using currency options as described in the previous section. Panel A of Table 5 reports the results from a panel estimation with country fixed effects.

[Table 5 about here.]

In the logic of the model, a relatively high foreign disaster risk exposure (high pJ^*) implies that the foreign currency appreciates in times of a disaster. Investing in such a currency provides insurance in bad times, and interest rates are thus low. The model therefore suggests that interest rates should decrease when disaster risk exposures increase. This is what we find in the data. Empirically, the slope coefficient in the regression above is negative and significant, equal to -1.9 over the whole sample. The results are not only driven by the 2008 crisis. The slope coefficient is similar when excluding the fall of 2008, and it is also negative and significant, albeit not as large, in the pre-2008 sample. In unreported results, we run similar tests at the country-level: the slope coefficients are negative in eight of our nine countries and significantly so in five of them. The results are also similar across currency portfolios. As the model suggests, higher disaster risk goes in hand with lower interest rates.

4.2 Disaster Risk and Exchange Rate Changes

In the model, the change in the exchange rate (measured in U.S. dollars per foreign currency) is given by the ratio of the home to foreign SDFs, as in Equation (1). In theory, the changes in exchange rates therefore reflects Gaussian shocks, as well as large, but rare jumps. If the domestic SDF shock is larger than the foreign one, the domestic currency appreciates (i.e., s decreases). For the sake of clarity, we have assumed that the Gaussian shocks are independent from the random variables that govern the impact (J, J^*) and the probability (p) of disaster. As already noted, however, in disaster risk models featuring Epstein-Zin preferences (e.g., Wachter, 2013), some Gaussian shocks are inherently the product of changes in disaster probabilities.

In the data, Gaussian and non-Gaussian variables may be correlated, and the realized changes in exchange rates, although driven most periods by their Gaussian shocks, may be correlated to the relative disaster risk exposures. To test this mechanism, we thus run the following contemporaneous regression between exchange rate changes and the changes in relative disaster risk exposures:

$$\Delta s_{j,t+1} = \alpha_j + \beta(\widehat{\Delta p J_{t+1}} - \widehat{\Delta p J^*_{j,t+1}}) + \epsilon_{j,t+1}.$$

Panel B of Table 5 reports the results from a panel estimation with country fixed effects. We note that the typical R^2 on those exchange rate regressions is on average 15%. It is in line with the explanatory power of the carry factor (the exchange rate of the high vs low interest rate currencies), suggesting that our disaster risk variables capture most of the relevant carry information (Verdelhan, 2014). The slope coefficient on the regression above is negative and significant in the full sample, with or without the fall of 2008. In a pre-2008 crisis sample, the slope coefficient is also negative and significant, and larger than in the full sample. We also obtain negative and significant slope coefficients when using the changes in relative disaster risk exposures instead of their levels. In unreported results, we run the same test at the country-level: the slope coefficient is negative in all nine countries. The slope coefficients are also negative and significant in portfolio-level tests.

This finding is consistent with the core premise of a disaster model of exchange rates like the

one presented in Farhi and Gabaix (2013): when the disaster risk of the domestic country increases (so that $\widehat{pJ_{t+1}}$ decreases), the domestic currency depreciates. In that model, there are two types of shocks: disaster shocks, that happen rarely (perhaps every few decades), and innovations to the probability and latent intensity of disaster shocks (i.e., innovation to pJ), that happen every period. Disaster shocks are priced and command a risk premium, whereas shocks to pJ are not priced, i.e. do not command a risk premium. Both affect the value of the exchange rate (s), but only disaster risk affects the risk premium on the exchange rate, hence the expected carry trade return. In the logic of that model, our estimates of pJ capture the innovation to the probability and (country-specific) latent intensity of disaster risk.

4.3   Disaster Risk and Equity Risk

We end this section with a novel empirical link between disaster risk and equity risk. Since our estimation recovers country-specific measures of disaster risk, we confront them to the option prices on the corresponding aggregate stock markets. We measure disaster risk in equity markets using risk reversals on stock market indices for the following countries: Australia, Canada, European Union, Japan, Norway, Sweden, Switzerland, and the United Kingdom. The sample window is January 2005 to October 2014 because of the data availability. The data come from Bloomberg and cover all the countries in our G10 sample, except New Zealand.

Options on equity indices are quoted in implied volatility for different levels of moneyness.[9] Let $iv_j^{EQ}(x)$ be the implied volatility on a stock market index for country j at a moneyness x. The equity market risk reversal for country j is defined by:

$$rr_j^{EQ}(y) = iv_j^{EQ}(1 - y) - iv_j^{EQ}(1 + y)$$

In what follows, we focus on option strikes that are 5% away from the money ($y = 5\%$). We

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[9]Option on equity indices are quoted in moneyness, whereas, as already noted, options on exchange rates are quoted in delta. An exchange rate option with a Δ equal to 0.1 corresponds approximately to a moneyness of 5%.

estimate the following contemporaneous regression of equity risk-reversals on disaster risk exposures:

$$rr_{j,t}^{EQ} = \alpha_j + \beta \widehat{pJ_{j,t}^*} + \epsilon_{j,t}.$$

Panel C of Table 5 reports the results from a panel estimation with country fixed effects. The regression coefficient is negative and significant, with or without the fall of 2008. It is also negative and significant, albeit smaller, in the pre-crisis period. In unreported results, the regression coefficient is negative and significant for half of the countries in our sample. Our findings imply that in periods where crash risk is high for a currency i, crash risk is also high in country i's stock market.

We have not derived equity returns nor equity derivatives in our model. We refer the reader to Farhi and Gabaix (2013) for such detailed analysis. In the logic of that paper, our results mean that when country resilience is low (because investors fear that export productivity will fall in a disaster), stock market resilience is also low (because investors fear that the stock market dividend will also greatly fall in a disaster). A contemporaneous regression of relative equity returns on relative risk exposures confirms the link between equity markets and the disaster risk exposure (a component of the log SDF). The results are reported in Panel D of Table 5. The link between equity returns and disaster risk exposure is only weakly significant and the explanatory power of disaster risk is limited. Yet, when disaster risk increases relatively more in the foreign country than in the U.S., the foreign equity markets offers lower returns than the U.S. stock market.

Our novel global disaster risk exposure uncovers new links between exchange rates, interest rates, and tail risk in equity markets: for a given country, when the (disaster) risk of a currency depreciation is high, its interest rate is high, its currency is depreciated, and tail risk in its stock market (as measured in equity risk-reversals) is high.

5 Additional Model Implications

In this section, we derive additional model implications on hedged returns and risk-reversals that are useful to interpret the literature on disaster risk and on the forward premium puzzle. We check our propositions through simulations and consider the impact of counterparty risk. Throughout, we now assume for simplicity that the disaster size stays constant within each month.

5.1 Hedged Carry Trade Returns

We first define hedged carry trades and then propose a closed-form expression for their expected returns.

5.1.1 Definition of Hedged Payoffs

In what follows, we drop the time subscripts for notational simplicity. Let Δ^P be a Black-Scholes put delta, $\Delta^P < 0$, and let K_{Δ^P} be the corresponding strike; $\Delta^P \in (-1, 0)$ is decreasing in the option strike. The return $X(K_{\Delta^P})$ to the hedged carry trade is the payoff of the following zero-investment trade: borrow one unit of the home currency at interest rate r; use the proceeds to buy $\lambda^P(K_{\Delta^P})$ puts with strike K_{Δ^P}, protecting against a depreciation in the foreign currency below K_{Δ^P}; and invest the remainder $\left(1 - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)$ in the foreign currency at interest rate $r^\star$. So the hedged return is given by:

$$X(K_{\Delta^P}) = \left(1 - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)e^{r^\star\tau}\frac{S_{t+\tau}}{S_t} + \lambda^P(K_{\Delta^P})\left(K_{\Delta^P} - \frac{S_{t+\tau}}{S_t}\right)^+ - e^{r\tau},$$

where the hedge ratio $\lambda^P(K_{\Delta^P})$ is given by:

$$\lambda^P(K_{\Delta^P}) = \frac{e^{r^*\tau}}{1 + e^{r^*\tau}P(K_{\Delta^P})}.$$

To summarize the notation: X is the carry trade return and X^e is its annualized expected value conditional on no disaster; $X(K_{\Delta^P})$ is the hedged carry trade return with strike K_{Δ^P}; $P(K_{\Delta^P})$ is

the home currency price of a put yielding $(K_{\Delta^P} - S_{t+\tau}/S_t)^+$ in the home currency; $X^e(K_{\Delta^P})$ is the annualized expected value of the hedged carry trade return conditional on no disaster; and E^{ND} denotes expectations under the assumption of no disaster:

$$X^e(K_{\Delta^P}) = \frac{E^{ND}[X(K_{\Delta^P})]}{\tau}.$$

5.1.2 A Simple and Intuitive Decomposition

Proposition 3 offers a closed-form formula for the hedged returns.

Proposition 3. *We assume that the disaster sizes (J, J^*) are constant between t and $t + \tau$ with $J > J^*$. Let Δ^P be a Black-Scholes put delta, $\Delta^P < 0$, and let K_{Δ^P} be the corresponding strike. We define:*

$$
\begin{aligned}
\beta &= n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)(1 + \Delta^P), \\
\gamma &= \big(1 + \Delta^P\big)\Delta^P \mathbb{N}^{-1}(-\Delta^P) - \big(2 + \Delta^P\big)n\big(\mathbb{N}^{-1}(-\Delta^P)\big),
\end{aligned}
$$

where $\mathbb{N}(\ )$ is the cumulative standard normal distribution and $n(\ )$ is the standard normal distribution. In the limit of small time intervals ($\tau \to 0$), the hedged carry trade expected return (conditional on no disasters) can be approximated by:

$$X^e(K_{\Delta^P}) = \big(1 + \Delta^P\big)\pi^G + \left(\beta\left(pJ + \frac{\pi^D \pi^G}{\sigma_h^2}\right) + \gamma\pi^G\right)\sigma_h\sqrt{\tau}, \tag{8}$$

where π^G is the Gaussian exposure, σ_h is the exchange rate volatility conditional on no disaster, and π^D is the disaster exposure.

Loosely speaking, in the limit of short time to maturity, the Black–Scholes delta of the put option has a simple interpretation: it is the probability that the put will be exercised. The first term in Equation (8) is thus intuitive: the further away from the money, the more depreciation risk the investor bears and the higher the expected return of the hedged carry trade. For example, take the

carry trade hedged with a put option at 10 delta. In the language of currency traders, this means that the strike is such that the Black-Scholes delta of the put is -0.10; thus the leading order of $X^e(K_{10^P})$ is equal to $0.9\pi^G$. Since the hedge uses a relatively deep-out-of-the-money put, investors bear 90% of the Gaussian risk. [10]

The second term in Equation (8) depends on a mixture of Gaussian and disaster parameters. Our model simulation, which is discussed in the next section, shows that, for the one-month maturity, it accounts for 1/5 to 1/3 of the hedged returns (depending on Δ^P) and is positive for any reasonable values of the model parameters. Proposition 3 thus leads to a simple upper bound for the Gaussian risk exposure and a lower bound for the disaster exposure:

$$\pi^G < \frac{X^e(K_{\Delta^P})}{(1+\Delta^P)} \text{ and } \pi^D > X^e - \frac{X^e(K_{\Delta^P})}{(1+\Delta^P)}. \tag{9}$$

Table 6 reports portfolio average currency excess returns that are unhedged or hedged at 10 delta, at 25 delta, and at-the-money for three portfolios. In each case, the table reports the mean excess return and its standard error, along with the corresponding Sharpe ratio for excess returns. As expected, hedging downside risks decreases average returns. Unhedged excess returns in high interest rate currencies are, again, equal to 5.4% on average (Panel I). A hedge at 10 delta protects the investor against large drops in foreign currencies, whereas a hedge at-the-money protects the investor against any foreign currency depreciation: the latter insurance is obviously more expensive because it covers more states of nature and thus leads to lower excess returns. Average excess returns hedged at 10 delta are 4.7% (Panel II), whereas average excess returns hedged at 25 delta and at-the-money are 3.5% and 2.1% (Panels III and IV). Including the Fall of 2008 in the sample leads to similar results: average excess returns hedged at 10 delta, 25 delta, and at-the-money are 3.9%, 2.9%, and 1.7%, respectively (not reported).

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[10] Jurek (2014) uses one-month currency excess returns hedged at- and out-of-the money to estimate the share of Gaussian and disaster risks. Our model provides a structural interpretation to his empirical experiment. When the investment horizon shrinks to zero, currency excess returns hedged out-of-the-money do not bear any disaster risk, but they offer biased estimates of the Gaussian risk exposure, since they bear 90% of the Gaussian risk at 10 delta, and 75% of the Gaussian risk at 25 delta. At the one-month horizon, however, our simulations show that the bias is important.

Using, for example, currency excess returns hedged at 25 delta leads to an upper bound for the Gaussian risk exposure of $3.5/0.75 = 4.7\%$ and to a lower bound bound for the disaster risk exposure of $5.4\% - 4.7\% = 0.7\%$. Likewise, hedged excess returns at-the-money imply an upper bound for the Gaussian risk exposure of 4.2% and a lower bound bound for the disaster risk exposure of 1.2%. These bounds are consistent with the estimates reported in Table 3.

This methodology, however, suffers from three weaknesses when compared to our benchmark estimation: (i) it only delivers bounds instead of point estimates, (ii) it delivers an average disaster risk exposure but not its time variation, and (iii) it relies on the estimation of two averages (hedged and unhedged excess returns), which are only known with large standard errors in small samples.

5.2 Risk-Reversals

We now turn to our model's implications for risk-reversals. Given $\Delta > 0$, we consider the corresponding Black-Scholes put delta, $\Delta^P = -\Delta$, as well as the Black-Scholes call delta, $\Delta^C = \Delta$. Risk-reversals are defined as the difference between the implied volatility at the Black-Scholes put delta and the implied volatility at the Black-Scholes call delta:

$$RR_\Delta = \sigma_{-\Delta} - \sigma_\Delta. \tag{10}$$

Risk-reversals are an appealing metric that highlights the key role of disaster risk in the price of options, posited in Propositions 4 and 5.

Proposition 4. *If there is no disaster risk : $RR_\Delta = 0$ for all Δ.*

A similar result was derived by Bates (1991) for equity options. In the presence of disaster risk, Proposition 5 identifies conditions under which we can simplify the expression for risk-reversals.

Proposition 5. *We assume that the disaster sizes* (J, J^*) *are constant between t and $t + \tau$. Given a Black-Scholes delta $\Delta > 0$, risk-reversals can be approximated in the limit of small time intervals ($\tau \to 0$) by:*

$$RR_\Delta = \frac{1 - 2\Delta}{n(\mathbb{N}^{-1}(\Delta))} \pi^D \sqrt{\tau}.$$

At short maturity, the risk–reversal is approximately proportional to the disaster exposure and increases approximately linearly with the distance to the money measured by Δ. When the foreign country is more exposed to disaster risk, both the interest rate difference and the short-maturity risk-reversal increase. These characteristics appear in our data set.

5.3   Simulations

Propositions 1, 3, and 5 are derived in the limit of small time intervals. We check their validity for one-day and one-month horizons by simulating a calibrated version of the model. The model relies on eight parameters: the disaster probability (p), the domestic and foreign disaster sizes (J and J^*), the domestic and foreign drifts (g and g^*) of the pricing kernels, the domestic and foreign volatilities (σ and σ^*) of the Gaussian shocks, as well as their correlation (ρ). The calibration thus relies on eight moments. The disaster probability is taken from Barro and Ursua (2008). The average domestic and foreign interest rates, the average domestic and foreign disaster sizes (scaled by p), the average currency excess returns, and the volatility of the bilateral exchange rate are all measured on the high interest rate currency portfolio during the period 1/1996–12/2011 excluding Fall 2008. The maximum Sharpe ratio is assumed to be 80%. The Online Appendix reports the parameters and simulation results.

The annualized, simulated unhedged returns are equal to 6.2% and 6% at the one-month and one-day horizons respectively, in line with the true value in the model (6%). Likewise, the simulated interest rates are equal to their calibrated targets. Proposition 1 thus delivers precise approximations of interest rates and average unhedged currency excess returns. These approximations are the only

ones needed to derive and interpret our main empirical results.

At the one-month horizon, the simulated hedged returns are equal to 4.3% at 10 delta, 3.2% at 25 delta, and 2.0% at-the-money. The approximations in Proposition 3 deliver hedged returns equal to 4.1% at 10 delta, 3.1% at 25 delta, and 1.9% at-the-money, close to the true values in the model. The approximations are the sum of two terms. The first term in Proposition 3, i.e., the fraction of the Gaussian risk exposure remaining, is equal to 2.70% at 10 delta, 2.25% at 25 delta, and 1.50% at-the-money. Thus, the second term, the unhedged component of the disaster exposure, cannot be neglected.

At the one-day horizon, the risk-reversal in the model is equal to 0.6% at 10 delta and 0.2% at 25 delta. The simulation shows that the approximation derived in Proposition 5 is close to the actual value; the approximated risk-reversal is equal to 0.7% at 10 delta and 0.2% at 25 delta. At the one-month horizon, however, the distance between the true and approximated risk-reversal is larger. The risk-reversal in the model is equal to 2.4% at 10 delta and 0.9% at 25 delta. The approximated risk-reversal is equal to 4% at 10 delta and 1.4% at 25 delta. Overall, the limit values derived in Propositions 3 and 5 appear as precise approximations at the one-day horizon. At the one-month horizon, however, their precision declines, especially for risk-reversals. We thus do not use these approximations to estimate the compensation for disaster risk. Yet, Propositions 3 and 5 remain useful to understand intuitively hedged currency excess returns and risk-reversals.

5.4 Counterparty Risk

All recent studies of disaster risk ignore counterparty risk. Yet, it is reasonable to think that the seller of a put might default with some probability ϕ if a disaster occurs, and that this risk is not fully-hedged by margin constraints. We are not able to measure default probabilities on option markets but obtain an order of magnitude of the potential impact on estimates of disaster risk exposure.

In the presence of counterparty risk, an agent engaging in hedged carry trade still bears some

disaster risk, even at short maturity. With probability ϕ, the agent is exposed to disasters and the compensation for the disaster risk is thus $\phi\pi^D$ and the expected excess return of the hedged carry trade is bounded below by $(1 + \Delta)\pi^G + \phi\pi^D$. According to Equation (9), in the limit of small time intervals, the disaster risk exposure is bounded by:

$$\pi^D > \frac{X^e - X^e(K_{\Delta^P})/(1 + \Delta^P)}{1 - \phi/(1 + \Delta^P)}.$$

For deep-out-of-the-money options ($\Delta = -0.1$), the lower bound for π^D that does not take into account counterparty risk must now be multiplied by $1/(1 - 1.1\phi)$. When $\phi = 0.1$, it is multiplied by 1.12; when $\phi = 0.25$, it is multiplied by 1.38. For at-the-money options ($\Delta = -0.5$), the adjustment is even larger: when $\phi = 0.1$, it is multiplied by 1.25; when $\phi = 0.25$, it is multiplied by 2.

Counterparty risk can substantially increase estimates of disaster risk exposure. Unfortunately, measuring expected default probabilities on option markets in disaster states is beyond the scope of this paper. The results above are only back-of-the-envelope estimates of the impact of counterparty risk. But they show that our estimates of disaster risk exposure certainly underestimate the true disaster risk exposure.

6 Literature Review

Our paper is related to three different literatures: the forward premium puzzle, disaster risk, and option pricing with jumps and stochastic volatility.

6.1 Forward Premium Puzzle

Since the pioneering work of Tryon (1979), Hansen and Hodrick (1980), and Fama (1984), many papers have reported deviations from the uncovered interest rate parity (UIP) condition. These deviations are also known as the forward premium puzzle. Recently, Lustig, Roussanov, and Verdel-

han (2011) build a cross-section of currency excess returns and show that it can be explained by covariances between returns and return-based risk factors. In large baskets of currencies, foreign country-specific shocks average out. Currency carry trades, defined as the difference in baskets of currency returns, are thus dollar-neutral and depend only on world shocks. In order to replicate the dynamics of exchange rates, Lustig, Roussanov, and Verdelhan (2011) show that SDFs must have a common component across countries, as well as heterogenous loadings on this common component. While these authors consider log-normal SDFs, Gavazzoni, Sambalaibat, and Telmer (2012) argue that SDFs should incorporate higher moments. Our paper builds on the disaster risk literature to satisfy these conditions.[11] World disaster risk is a common component of SDFs, but countries differ in their exposures to world disasters, which affect the higher moments of SDFs.

Taking their model to the data, Lustig, Roussanov and Verdelhan (2011) show that time-varying volatility in global equity markets accounts for the cross-section of forward discount-based currency portfolio returns. This volatility measure does not use any exchange rate or interest rate data, but illustrates the systematic risk of currency markets. During periods of high global volatility, high interest rate currencies tend to depreciate, while low interest rate currencies tend to appreciate. Menkhoff et al. (2012) find that a measure of global volatility obtained from currency markets also helps to explain the cross-section of interest rate-sorted currency portfolios.

How do these results relate to our paper? It turns out that large increases in global equity volatility corresponds to large increases in downside risk, and downside risk could as well account for the returns on the interest rate-sorted currency portfolios. Disentangling downside risk from volatility risk is not an easy task in a cross-sectional asset pricing experiment. To illustrate this

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[11]Other models replicate the forward premium puzzle. Using swap rates, exchange rate returns, and prices of at-the-money currency options, Graveline (2006) estimates a two-country term structure model that replicates the forward premium anomaly. Verdelhan (2010) uses habit preferences in the vein of Campbell and Cochrane (1999). Colacito (2008), Bansal and Shaliastovich (2012), and Colacito and Croce (2013) build on the long-run risk model pioneered by Bansal and Yaron (2004). Farhi and Gabaix (2013) propose a disaster risk explanation of the puzzle and the full term structure of interest rates, while Guo (2007) presents a disaster-based model with monetary frictions. Martin (2011) solves a two-country model with jumps, emphasizing the interaction between intratemporal and intertemporal prices. Gourio, Siemer, and Verdelhan (2013) study disaster risk in a two-country real-business cycle model. Della Corte, Ramadorai, and Sarno (2013) study the predictability of the option-implied volatility risk premia for exchange rate changes.

difficulty, the Online Appendix reports asset pricing tests on the six portfolios of Lustig et al. (2011) obtained with two risk factors: the average excess returns of a U.S. investor on currency markets (denoted RX, as in the two papers above) and the risk-reversals at 25 delta on S&P 500 Index options (denoted RR). The U.S. S&P 500 Index options are used to measure global disaster equity risk because of the lack of data on out-of-the-money equity options in other countries in the sample.

Risk-reversals are significantly priced in the cross-section of carry trade excess returns. Both factors help to explain more than 90% of the cross-section of average excess returns. Loadings on the dollar risk factor are close to 1 and do not account for the cross-section of portfolio returns. Loadings on risk-reversals, however, differ markedly across portfolios: they range from 0.87 to -0.96. Unsurprisingly, the same pattern characterizes our smaller set of countries and portfolios (for which betas vary from 0.81 to -0.76). High interest rate currencies tend to depreciate during bad economic periods, when risk-reversals are high, while low interest rate currencies tend to appreciate during those times. Lettau, Maggiori, and Weber (2013) report further evidence of downside risk in the cross-section of currency, equity, and commodity returns. Instead, we estimate a structural model on option prices to disentangle time-varying volatility from disaster risk exposure.

6.2 Disaster Risk

Our paper also relates to a recent literature using options to investigate the quantitative importance of disasters in currency markets.[12] Bhansali (2007) was the first to document the empirical properties of hedged carry trade strategies. Brunnermeier, Nagel, and Pedersen (2008) show that risk reversals increase with interest rates. In their view, the crash risk of the carry trade is due to a possible unwinding of hedge fund portfolios. This is consistent with one interpretation of disasters. Jurek (2014) provides a comprehensive empirical investigation of hedged carry trade strategies.

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[12]A large literature focuses instead on equity and bond markets: see Duffie, Pan, and Singleton (2000), Ait-Sahalia, Wang, and Yared (2001), Pan (2002), Liu, Pan, and Wang (2005), Gourio (2008), Barro and Ursua (2009), Santa-Clara and Yan (2010), Backus, Chernov, and Martin (2011), Bollerslev and Todorov (2011), Gabaix (2012), Julliard and Ghosh (2012), Bates (2012), Kelly and Jiang (2012), Siriwardane (2013), Martin (2013), Wachter (2013), Tsai and Wachter (2014), Kelly and Jiang (2014), Gao and Song (2013), Wachter and Seo (2015), and Wachter and Kilic (2014). Tsai and Wachter (2015) provide a recent and excellent survey.

Our approach differs in several dimensions. First, our model-based empirical strategy leads to a structural interpretation of the results. Second, the model allows us to use a variety of option strikes, including more-liquid at-the-money options, in order to disentangle Gaussian and disaster risk exposure. Third, we take into account the time-varying volatilities in currency markets. Using at-the-money options, Burnside et al. (2011) also find that disaster risk can account for the carry trade premium, where disaster risk comes in the form of a high value of the SDF rather than large carry trade losses. In contrast to our approach, in their framework the only source of risk priced in carry trade returns is disaster risk and they only consider at-the-money options. Our model shows in closed-form that average hedged excess returns at-the money are not zero in the presence of Gaussian risk. All those papers focus on the pre-crisis period, while our paper uncovers key differences in the post-crisis period. Our paper complements Du (2013) who studies consumption disasters in currency markets. Our two models share the ability to generate frequent sign switches in the risk-neutral skewness of currency returns, a feature necessary to replicate option smiles. Our model differs by allowing both Gaussian and disaster risk to potentially account for currency risk premia. Our estimation is run jointly on all currency pairs in order to take into account the common parameters introduced by a common base currency, the U.S. dollar. Using an approach similar to ours, i.e., matching model-based currency option prices to their empirical counterparts, Jurek and Xu (2014) recently estimates a model that includes both country-specific and global disasters and a more involved characterization of jumps. Our model is arguably more parsimonious, easier to interpret, and delivers comparable average option pricing errors, of the same order of magnitude as the option bid-ask spreads. Jurek and Xu (2014) conclude that higher-order moments of the pricing kernel innovations account on average for only 15% of the carry trade risk premium. As we shall see, this finding is reasonably close to ours: we find that the average disaster risk premium is close to a third of the carry trade risk premium. Finally, our paper is related to recent work by Chernov, Graveline, and Zviadadze (2012), who study daily changes in exchange rates and at-the-money implied volatilities. Unlike us, however, they fully parametrize a law of motion for the stochastic discount factor using a rich model specification that includes stochastic volatility and jumps in vari-

ance for the gaussian risk, as well as jumps for the crash risk. They find that jump risk accounts for 25% of currency risk and show that many jumps in levels are related to macroeconomic news, while jumps in volatilities are not. We do not specify the law of motion of the parameters, which therefore change freely at the monthly frequency, allowing us to uncover a clear structural break in the Fall of 2008.

Our estimates of the compensation for disaster risk exposure and carry trade losses during Fall 2008 are broadly consistent with the results in the macro-finance literature on disaster risk, notably the findings and calibration of Barro (2006) and Barro and Ursua (2008, 2009). When a disaster occurs in our model, the SDF is multiplied by an amount J. The model of Farhi and Gabaix (2013) relates this amount to more primitive economic quantities. In that model, J equals $B^{-\gamma}F$, where $B^{-\gamma}$ is the growth of real marginal utility during a disaster and F is the growth of the value of one unit of the local currency in terms of international goods during the same disaster. Hence, the disaster risk exposure is in that model:

$$\overline{\pi}^D = \overline{pE[J]}_L - \overline{pE[J]}_H = \overline{pE[B^{-\gamma}F]}_L - \overline{pE[B^{-\gamma}F]}_H,$$

where the subscripts L and H refer to low and high interest rate countries. Therefore, the disaster risk exposure depends on the probability of disasters p, the relative value of the SDF $B^{-\gamma}$, and the payoff of the carry trade in disasters through the sufficient statistic $\overline{pE[B^{-\gamma}F]}_L - \overline{pE[B^{-\gamma}F]}_H$.

Using the episode of Fall 2008 to calibrate the value of $\overline{F}_L - \overline{F}_H$ and assuming away a potential correlation between $B^{-\gamma}$ and $\overline{F}_L - \overline{F}_H$ sheds some light on the typical value of $pB^{-\gamma}$. This exercise should be viewed as a back-of-the-envelope calculation rather than a rigorous estimate, since the inference of $\overline{F}_L - \overline{F}_H$ relies on a single disaster. Moreover, it does not take into account the full path to recovery and, as Gourio (2008) shows, might overestimate the impact of disasters. With this caveat in mind, a value for $\overline{F}_L - \overline{F}_H$ of 20% (in line with the cumulative loss of the high interest rate portfolio in 2008) implies a value of $\overline{pE[B^{-\gamma}]}$ equal to 10% to generate a disaster risk exposure $\overline{\pi}^D$ of 2%, as in the currency option data.

To check the order of magnitude of this implied $\overline{pE[B^{-\gamma}]}$, we refer to Barro and Ursua (2008), who use long samples of consumption series for a large set of countries to estimate disaster sizes and probabilities.[13] They estimate a probability of disasters p equal to 3.63%. A coefficient of relative risk aversion γ equal to 3.5 rationalizes the equity premium; it implies that $\overline{E[B^{-\gamma}]} = 3.88$, leading to a value of $\overline{pE[B^{-\gamma}]}$ equal to 14%, which is close to our estimate. In other words, Barro and Ursua's (2008) value of 14% for $\overline{pE[B^{-\gamma}]}$ and a carry trade loss of 20% during disasters led to a disaster risk exposure of $0.14 \times 0.2 = 2.8\%$. Therefore, our estimates over the 1996 to 2014 period (2.3%) are consistent with Barro and Ursua's (2008) findings.

6.3 Option Pricing

A related literature studies high-frequency data and option pricing with jumps, following pioneering work by Merton (1976) in the context of equity options. Borensztein and Dooley (1987) extend the use of models with jumps to currency options. Bates (1996a, 1996b) studies the role of exchange rate jumps in explaining currency option smiles. Bates (2012) shows that volatility smirk implications of some stochastic volatility models without jumps are identical to various models with jumps, for strike prices sufficiently close to the money. Carr and Wu (2007) find great variations in the riskiness of two currencies (yen and British pound) against the U.S. dollar, and they relate it to stochastic risk premia. Campa, Chang, and Reider (1998) document similar results for some European cross-rates. Bakshi, Carr, and Wu (2008) find evidence that jump risk is priced in currency options. However, they consider jumps that occur at a high frequency, whereas the disasters we have in mind are of very low frequency; in Barro and Ursua (2008), disasters happen every 30 years. As a result, the economic analysis and our econometric technique are very different from the traditional option pricing literature. Our focus is on the macro-finance explanations of currency risk.

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[13]Note, however, that interpreting our pricing kernel strictly as a simple function of consumption growth would open a large debate that is beyond the scope of this paper. Constant relative risk aversion and complete markets imply, for example, a very high correlation between consumption growth and exchange rates, a high correlation that is not evident in the data (Backus and Smith, 1993).

7 Conclusion

Our goal in this paper is to provide a simple, real-time, model-based estimation of the compensation for world disaster risk. We achieve this goal using currency options. The Fall of 2008 appears as a turning point in currency option markets: option smiles are fairly symmetric before the financial crisis; post-crisis, they are clearly asymmetric, and those asymmetries depend on the level of interest rates. The model interprets the data in terms of disaster risk. High (low) interest rate currency options reflect the risk of large depreciations (appreciations) during bad economic times. The model estimation shows that while the compensation for global disaster risk was low before the crisis, it remains an order of magnitude higher afterwards. The disaster risk premium accounts for more than a third of the carry trade risk premium in advanced countries over our sample. Disaster risk offers a potential interpretation to the cross-sectional and time-series variation of interest rates and exchange rates.

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Table 1: Average Risk-Reversals Before, During, and After the 2008 Crisis

Portfolios	1	2	3
Panel I: 1/1996–08/2014			
Mean	−0.45	0.70	1.39
	[0.26]	[0.22]	[0.30]
Panel II: 1/1996–08/2014 (excl. 09/2008–12/2008)			
Mean	−0.39	0.67	1.30
	[0.25]	[0.20]	[0.28]
Panel III: 1/1996–8/2008			
Mean	−0.71	0.19	0.58
	[0.23]	[0.11]	[0.14]
Panel IV: 9/2008–12/2008			
Mean	−3.89	2.41	6.43
	[2.37]	[1.31]	[1.85]
Panel V: 1/2009–08/2014			
Mean	0.35	1.75	2.94
	[0.45]	[0.39]	[0.46]

Notes: This table reports portfolio average risk-reversals at 10 delta over different subsamples (Panels I to V). Risk-reversals are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains risk-reversals from the lowest interest rate currencies while the last portfolio contains risk-reversals from the highest interest rate currencies. Risk-reversals are reported in percentages. The standard errors, reported between brackets, are obtained by bootstrapping both the time series using a block bootstrap of 10 months (1 month during the crisis) and the cross-section of countries. Data are monthly, from J.P.Morgan. The sample period is 1/1996 to 08/2014.

Table 2: Exchanges Rate Changes, Risk-Reversals, and Currency Excess Returns

Portfolios	1	2	3
	Panel I: Exchange Rates		
Mean	0.28	0.47	1.25
	[1.71]	[1.82]	[2.08]
	Panel II: Interest Rates		
Mean	−1.90	0.14	2.32
	[0.38]	[0.28]	[0.29]
	Panel III: Risk-Reversals 10 Delta		
Mean	−0.45	0.70	1.39
	[0.26]	[0.22]	[0.30]
	Panel IV: Risk-Reversals 25 Delta		
Mean	−0.23	0.40	0.76
	[0.14]	[0.12]	[0.16]
	Panel V: Excess Returns		
Mean	−1.10	1.10	4.27
	[1.87]	[1.88]	[2.15]
Sharpe Ratio	−0.14	0.13	0.41

Notes: This table reports portfolio average changes in exchange rates, interest rates, risk-reversals, as well as average currency excess returns. Countries are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains the lowest interest rate currencies while the last portfolio contains the highest interest rate currencies. The table reports the mean excess return and its standard error, along with the corresponding Sharpe ratio for excess returns. The mean and standard deviations for the exchange rates, the interest rates, and the excess returns are annualized (multiplied respectively by 12 and $\sqrt{12}$). The Sharpe ratio corresponds to the ratio of the annualized mean to the annualized standard deviation. The standard errors, reported between brackets, are obtained by bootstrapping both the time series using a block bootstrap and the cross-section of countries. The block sizes are 10 months. Data are monthly, from J.P.Morgan. The sample period is January 1996 to August 2014.

Table 3: Average Disaster Risk Exposure

	$\overline{\pi_D}$	$\overline{X}$	Disaster Share
	Panel I: 1/1996–08/2014		
Mean	2.29	4.27	53.46
	[0.66]	[2.15]	[30.86]
	Panel II: 1/1996–08/2014 (excl. 09/2008–12/2008)		
Mean	2.15	5.38	40.03
	[0.68]	[2.24]	[30.53]
	Panel III: 1/1996–08/2008		
Mean	0.46	3.60	12.74
	[0.22]	[2.80]	[7.74]
	Panel IV: 1/2009–08/2014		
Mean	6.00	9.43	63.65
	[1.40]	[6.07]	[23.05]

Notes: This table reports the estimates of disaster risk exposure $(\overline{\pi^D})$ and average currency carry trade excess return $(\overline{X})$, as well as their ratio, over different time-windows.[14] Estimations at the country level are aggregated into portfolios and the table reports the average estimates obtained for the portfolio of high interest rate currencies presented in Table 2. Standard errors are obtained by bootstrapping using a block bootstrap. Spot and forward exchange rates are from Datastream, while currency options are from J.P.Morgan. Data are monthly. The sample period is January 1996 to August 2014.

Table 4: Pricing Errors

	Pricing Error	Bid-Ask Spread
Panel I: 10 Delta Put		
Mean	0.06	0.62
	[0.04]	[0.30]
Panel II: 25 Delta Put		
Mean	0.04	0.58
	[0.03]	[0.30]
Panel III: At-The-Money		
Mean	0.02	0.40
	[0.01]	[0.16]
Panel IV: 25 Delta Call		
Mean	0.04	0.74
	[0.02]	[0.37]
Panel V: 10 Delta Call		
Mean	0.08	0.88
	[0.05]	[0.42]

Notes: This table reports the RMSE of the pricing errors (left) and the RMSE of the bid-ask spreads (right), obtained for the portfolio of high interest rate currencies presented in Table 2 for each strike (Panels I to V). Standard errors are obtained by bootstrapping using a block bootstrap. Spot and forward exchange rates are from Datastream, while currency options are from J.P.Morgan and bid-ask spreads are from Bloomberg. Data are monthly. The sample period is January 1996 to August 2014.

Table 5: Disaster Risk Across Markets: Contemporaneous Regressions

	With Fall 2008	Without Fall 2008	Pre 2008
	Panel A: Interest Rates		
$pJ*$	-1.88	-1.93	-0.93
	[0.37]	[0.38]	[0.21]
N	1,980	1,944	1,332
R^2	49.62	49.46	66.64
	Panel B: Exchange Rate Changes		
$\Delta\Pi_D$	-485.34	-502.94	-791.57
	[60.08]	[61.74]	[81.21]
N	1,929	1,893	1,281
R^2	14.76	14.67	16.82
	Panel C: Equity Risk Reversals		
$pJ*$	-3.47	-3.50	-3.01
	[0.97]	[1.01]	[1.21]
N	808	781	259
R^2	6.02	6.54	8.77
	Panel D: Equity Returns		
$\Delta\Pi_D$	101.98	74.64	38.11
	[37.55]	[41.91]	[92.10]
N	1,817	1,785	1,219
R^2	0.93	0.86	0.76

Notes: Panel A reports results of the linear regression: $r_{j,t} = \alpha_j + \beta\widehat{pJ^*_{j,t}} + \epsilon_{j,t}$, where $\widehat{pJ^*_{j,t}}$ is estimated using currency options. Panel B reports results of the linear regression: $\Delta s_{j,t+1} = \alpha_j + \beta(\widehat{\Delta pJ_{t+1}} - \widehat{\Delta pJ^*_{j,t+1}}) + \epsilon_{j,t+1}$. Panel C reports results of the linear regression: $RR^{Equity}_{j,t} = \alpha_j + \beta\widehat{pJ^*_{j,t}} + \epsilon_{j,t}$, where $RR^{Equity}_{j,t}$ represents the risk-reversal on the aggregate stock market in country j. Panel D reports results of the linear regression: $r^{EQ}_{j,t+1} - r^{EQ}_{t+1} = \alpha_j + \beta(\widehat{\Delta pJ_{t+1}} - \widehat{\Delta pJ^*_{j,t+1}}) + \epsilon_{j,t+1}$, where r^{EQ}_j denotes the equity return in foreign currency in country j, while r^{EQ} and pJ_{t+1} denote respectively the equity return and disaster risk exposure in the U.S. Exchange rates are in U.S. dollars per unit of foreign currency. Foreign interest rates are estimated from forward and spot exchange rates assuming that covered interest rate parity holds. Country disaster risk exposures $\widehat{pJ^*}$ are estimated using the procedure described in Section 3 of the paper. All left-hand side variables are in percentage points. Standard errors are computed using a Newey-West procedure; they do not take into account the uncertainty stemming from the estimation of the country disaster risk exposure $\widehat{pJ^*}$. Panel estimations include country fixed effects. The full period is 1/1996 to 8/2014. Equity risk-reversal series start in 1/2005. The first column includes the fall of 2008, while the second column excludes it (9/2008 to 12/2008); the third column focuses on the pre-2008 sample (up to 8/2008).

Table 6: Hedged Currency Excess Returns

Portfolios	1	2	3
	Panel I: Excess Returns		
Mean	−1.03	1.91	5.38
	[1.80]	[1.85]	[2.24]
Sharpe Ratio	−0.13	0.24	0.55
	Panel II: Excess Returns Hedged at 10 Delta		
Mean	−1.75	1.04	4.73
	[1.72]	[1.80]	[2.19]
Sharpe Ratio	−0.24	0.14	0.53
	Panel III: Excess Returns Hedged at 25 Delta		
Mean	−1.65	0.66	3.53
	[1.43]	[1.56]	[1.94]
Sharpe Ratio	−0.26	0.10	0.45
	Panel IV: Excess Returns Hedged ATM		
Mean	−1.28	0.25	2.10
	[0.99]	[1.13]	[1.47]
Sharpe Ratio	−0.28	0.05	0.36

Notes: To illustrate Proposition 3, which characterizes the expected returns in a sample without disasters, this table reports portfolio average currency excess returns that are unhedged or hedged at 10 delta, at 25 delta, and at-the-money for three portfolios. Countries are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains the lowest interest rate currencies while the last portfolio contains the highest interest rate currencies. The table reports the mean excess return and its corresponding Sharpe ratio. The mean and standard deviations are annualized (multiplied respectively by 12 and $\sqrt{12}$). The Sharpe ratio corresponds to the ratio of the annualized mean to the annualized standard deviation. Standard errors are obtained by bootstrapping. Data are monthly, from J.P.Morgan. The sample period is January 1996 to August 2014, excluding the Fall of 2008.
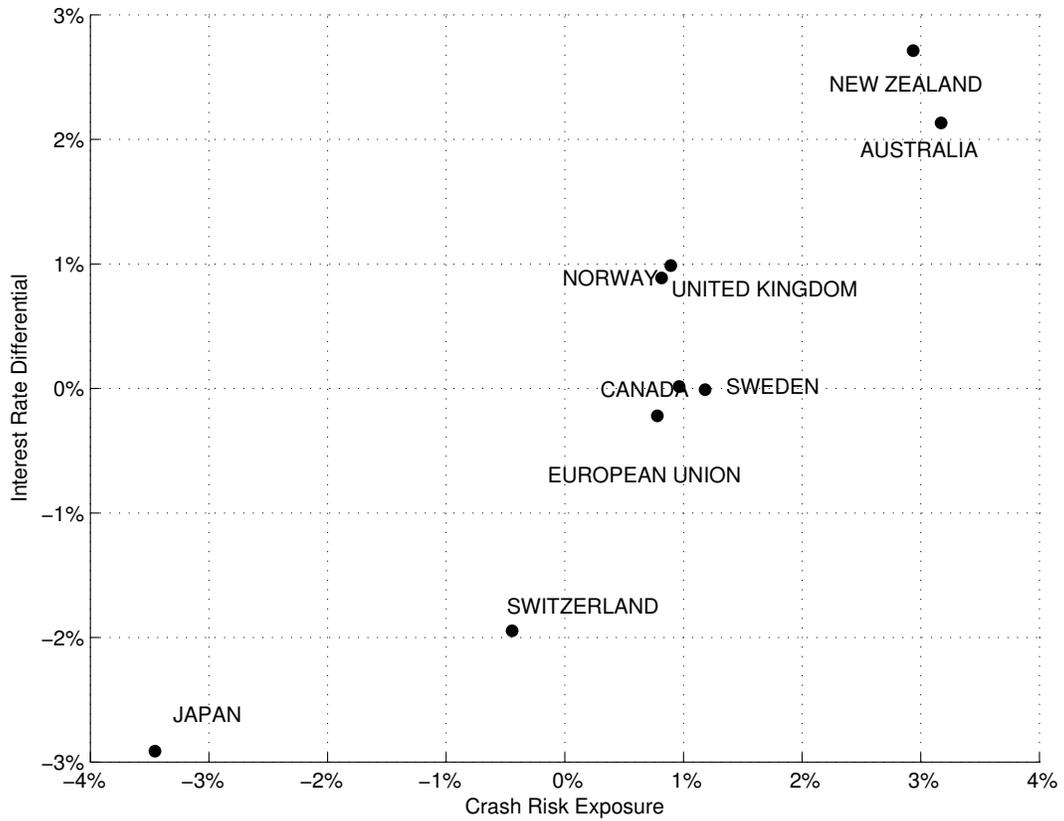
Figure 1: Average Compensation for Disaster Risk Exposure and Average Interest Rates

This figure reports the average compensation for disaster risk exposure and the average interest rate differential (vis-à-vis the U.S.) for each country. Interest rates and risk exposures are reported in percentage points per annum. Spot and forward exchange rates are from Datastream, while currency options are from J.P.Morgan. Data are monthly. The sample period is January 1996 to August 2014.
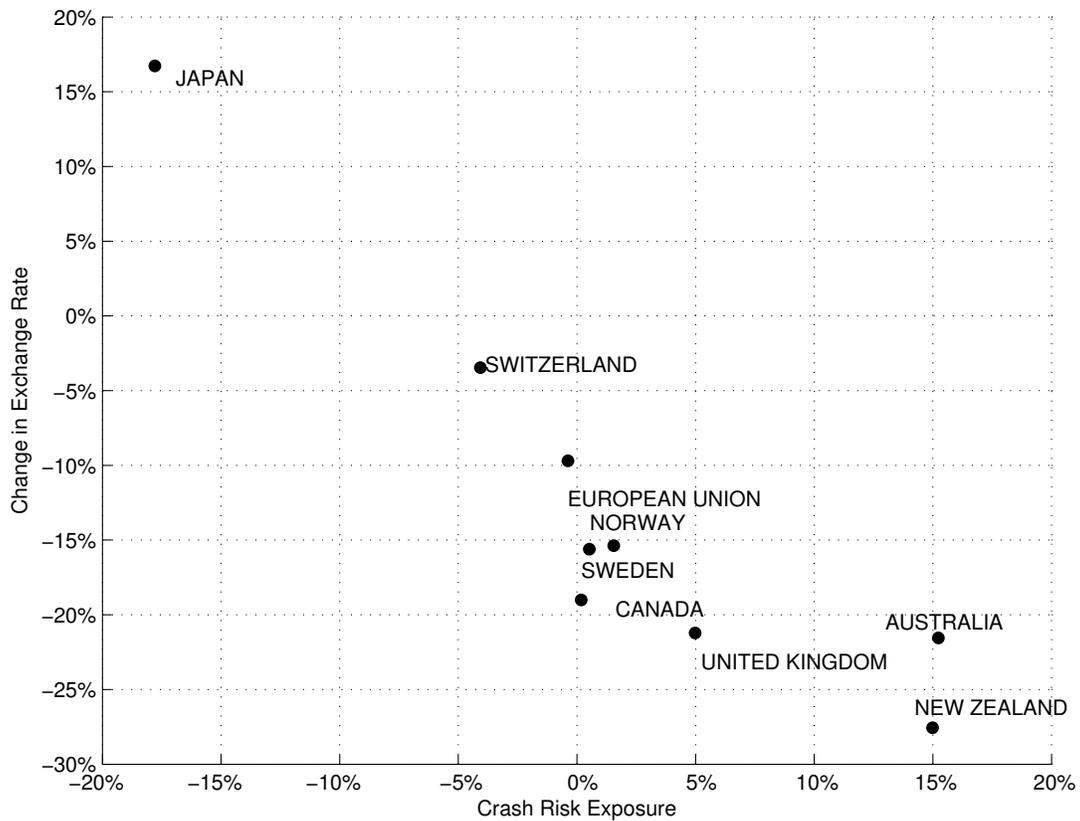
Figure 2: Disaster Risk Exposure and Changes in Exchange Rates During the Crisis

This figure reports the average estimated compensation for disaster risk exposure and the cumulative percentage change in exchange rate for each country from September 2008 to January 2009. Spot and forward exchange rates are from Datastream, while currency options are from J.P.Morgan. Data are monthly.
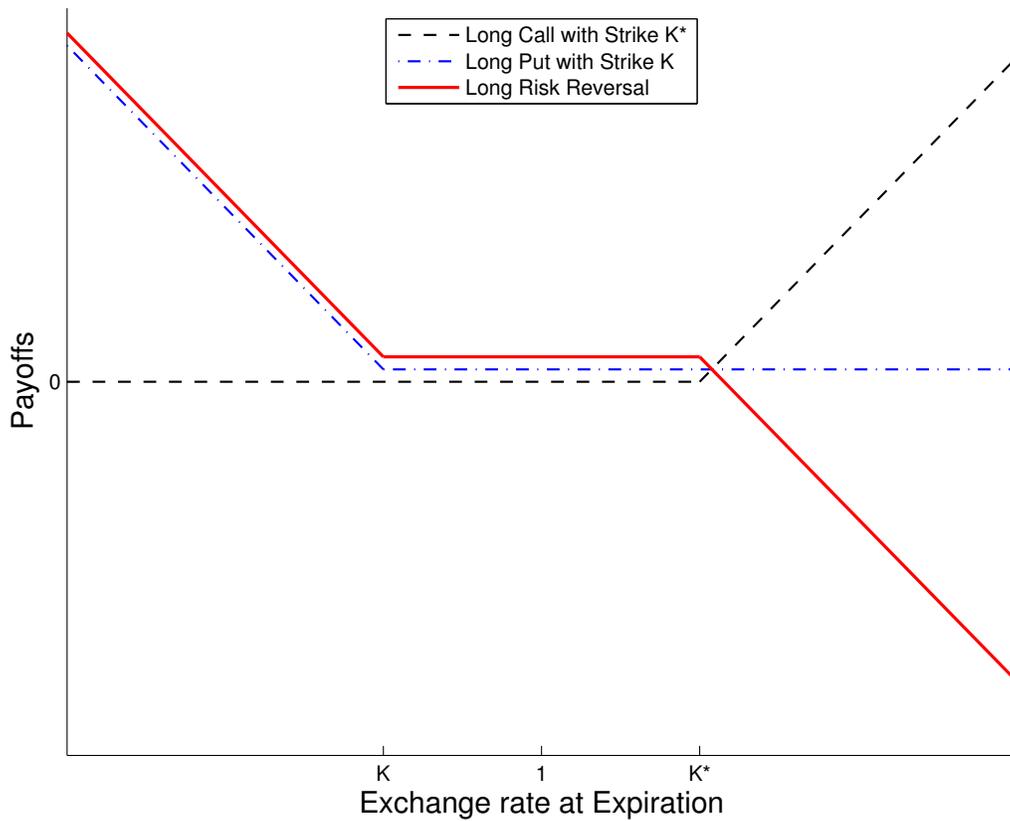
Figure 3: Option Payoffs

This figure presents the payoffs of three option investments as a function of the underlying asset at the expiration date. The underlying asset at the expiration date is normalized by the current forward price of the underlying asset. The three strategies consist of (i) buying an out-of-the money call (with strike $K^\star$); (ii) buying an out-of-the-money put option (with strike K); and (iii) a risk-reversal that corresponds to selling an out-of-the-money call (with strike $K^\star$) and simultaneously buying an out-of-the-money put (with strike K).
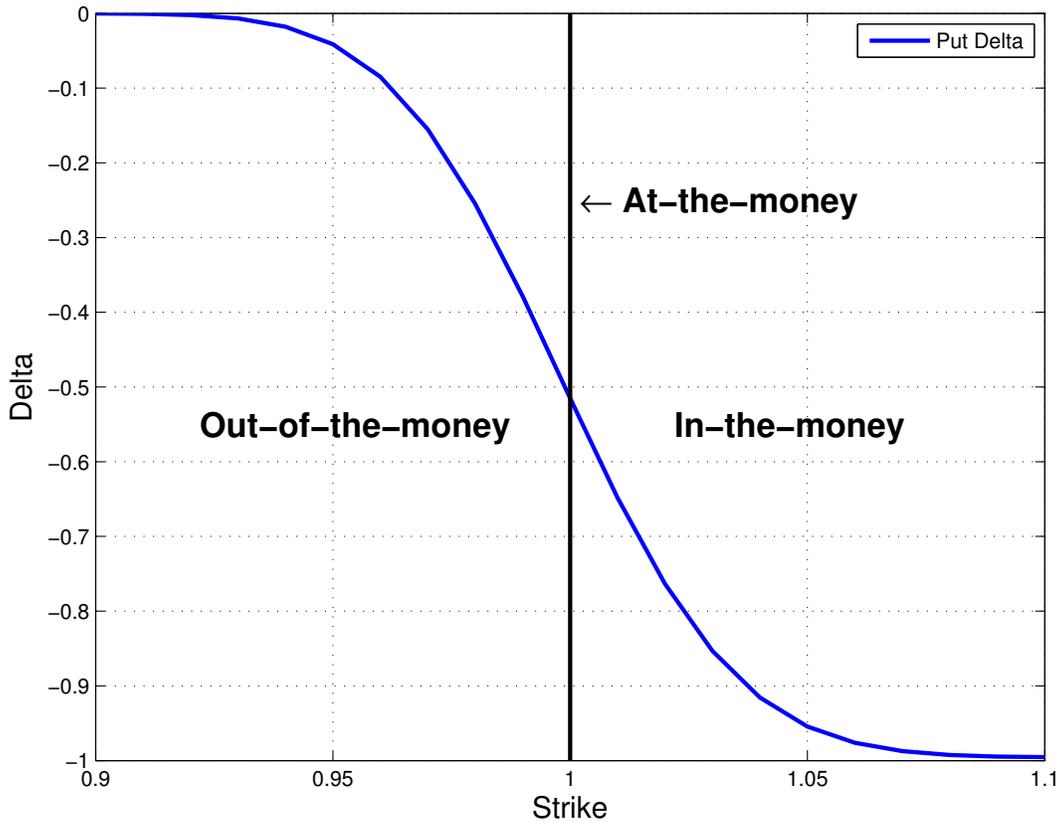
Figure 4: Option Deltas

This figure presents the deltas of put options as a function of their strikes. The strikes are normalized by the current forward price of the underlying asset. The delta of an option is defined as the rate of change of the option price with respect to the price of the underlying asset. The delta of a put varies between 0 for the most deep out-of-the-money options and -1 for the most deep in-the-money options. The figure is computed using the Black–Scholes formula for a currency put option with a one-month maturity, an annualized implied volatility of 10%, and foreign and domestic interest rates both set equal to 4% per annum.
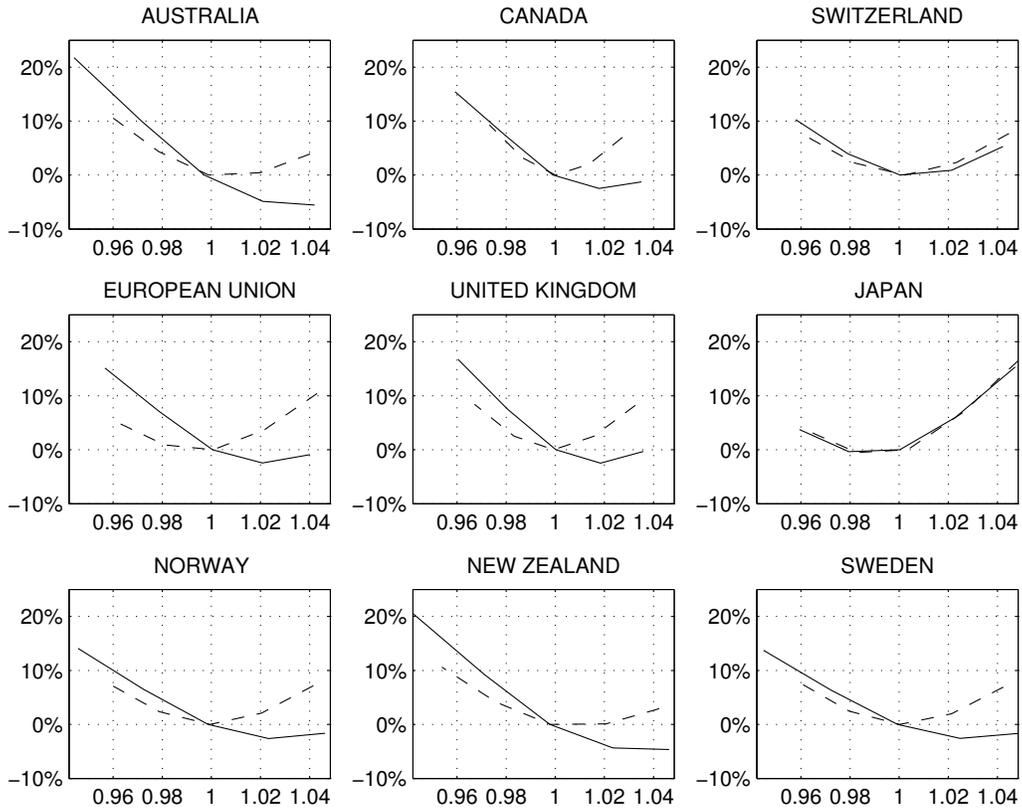
Figure 5: Average Currency Option Smiles Pre- and Post-Crisis

This figure presents the average quoted implied volatilities during the pre-crisis period (1/1996–08/2008, dotted line) and during the post-crisis period (1/2009–08/2014, full line) as a function of their strikes. To maintain comparability across currencies and periods, the implied volatilities at different strikes are scaled by the average implied volatility of at-the-money options during the corresponding period. The quoted strikes are normalized by the spot exchange rate. Spot and forward exchange rates are from Datastream, while currency options are from J.P.Morgan. Data are monthly.
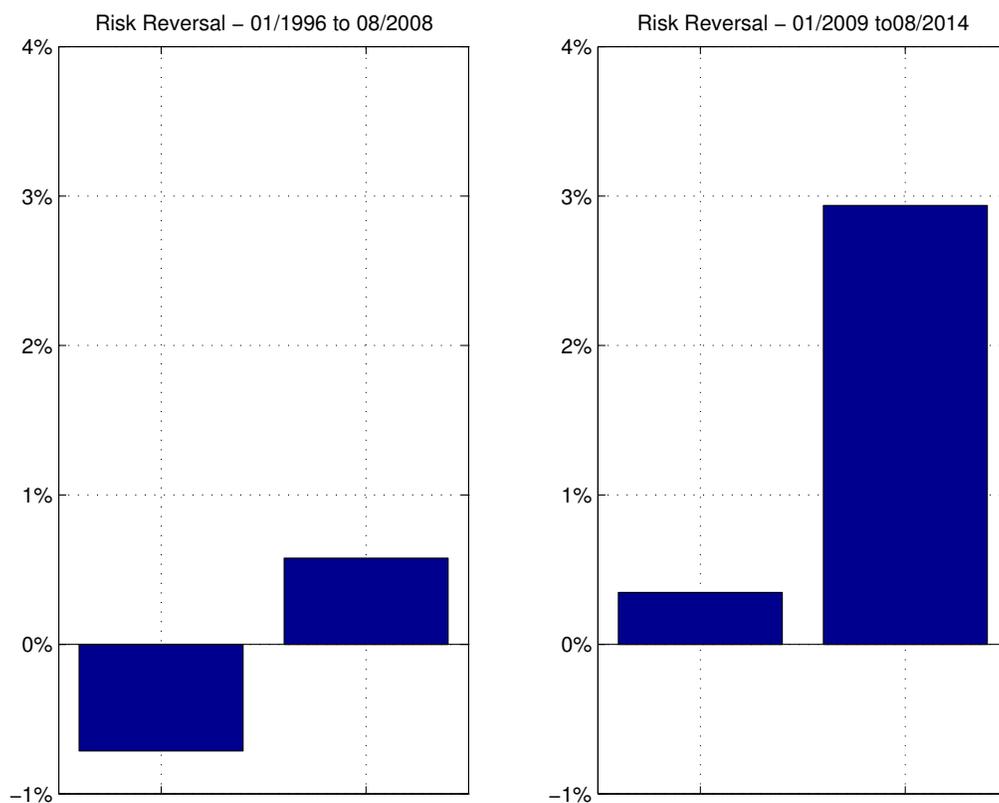
Figure 6: Average Risk Reversals Pre- and Post-Crisis

This figure presents the average risk-reversals at 10 delta of high and low interest rate currencies over two periods: 1/1996–8/2008 on the left panel, and 1/2009–08/2014 on the right panel. Risk-reversals are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains risk-reversals from the lowest interest rate currencies while the last portfolio contains risk-reversals from the highest interest rate currencies. In each panel, the left bar corresponds to the portfolio of low interest rate currencies, while the right bar corresponds to the portfolio of high interest rate currencies. Data are monthly, from J.P.Morgan.
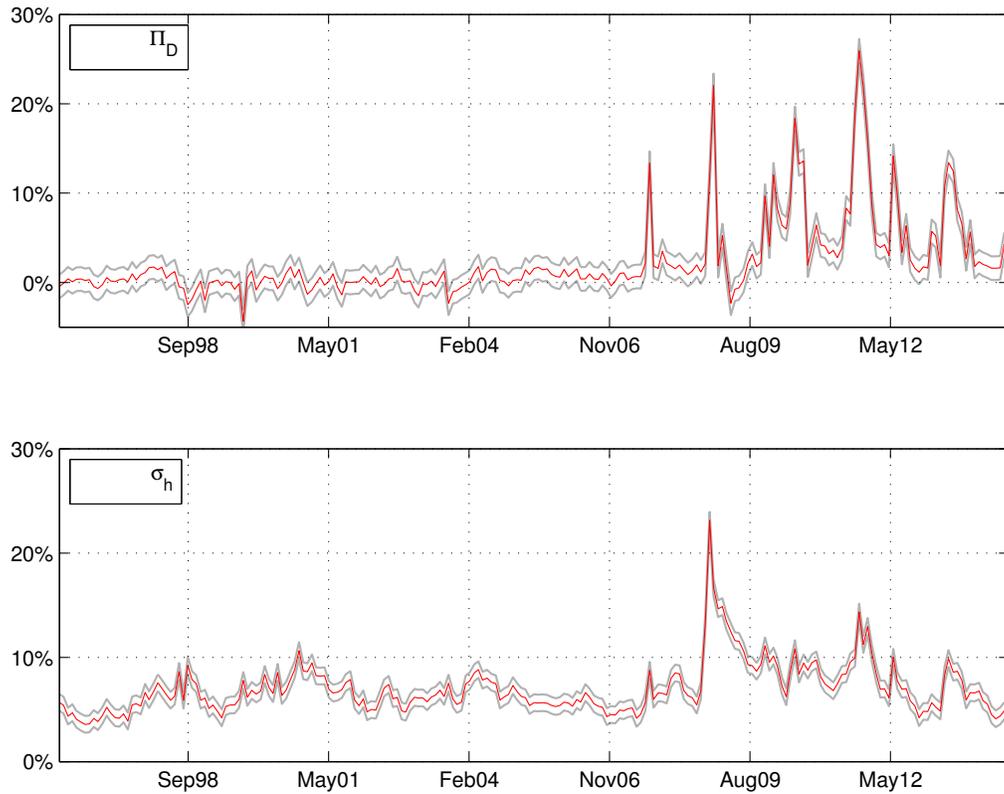
Figure 7: Time Series of Disaster Risk Exposure and Expected Currency Volatility

This figure presents the time series estimates of the average disaster risk exposure (top panel) and of the average volatility parameter (bottom panel) among the currencies in the high interest rate portfolio. The shaded area corresponds to two standard errors above and below the mean estimates. The standard errors are obtained by bootstrapping both the time series (using a block bootstrap of 10 months) and the cross-section of countries. Spot and forward exchange rates are from Datastream, while currency options are from J.P.Morgan. Data are monthly. The sample period is January 1996 to August 2014.